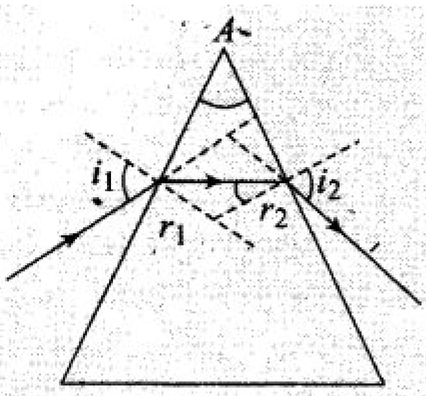
Chapter 9 - Ray Optics and Optical Instruments

Multiple Choice Questions (MCQs)

Single Correct Answer Type
Question 1. A ray of light incident at an angle d on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is mad? of a material of refractive index 1.5, the angle of incidence is
(a) 7.5° (b) 5° (c) 15° (d) 2.5°
Solution: (a)
Key concept:
In thin prisms, the distance between the refracting surfaces is ineligible and the angle of prism (A) is very small. Since A = $r_1 + r_2$, therefore if A is small then both r_1 , and r_2 are also small, and the same is true for $i_1$and i_2.



According to Snell's law, $1 \cdot \sin i_1 = \mu \cdot \sin r_1 \Rightarrow i_1 = \mu \cdot r_1$
Also, $1 \cdot \sin i_2 = \mu \cdot \sin r_2 \Rightarrow i_2 = \mu \cdot r_2$
Therefore, deviation, $\delta = (i_1 - r_1) + (i_2 - r_2)$
$\Rightarrow \quad \delta = (i_1 + i_2) - (r_1 + r_2) = (r_1 + r_2)(\mu - 1)$
$\Rightarrow \quad \delta = A(\mu - 1)$

Since, deviation $\delta = (\mu - 1)A$
$= (1.5 - 1) \times 5° = 2.5°$

The angle of the prism is 5°. The ray emerges from refracting face of a prism normally.
Then, $i_2 = r_2 = 0$
As $A = r_1 + r_2 \Rightarrow r_i = A$ or $r_1 = 5°$
But $i_1 = \mu \cdot r_1 = \frac{3}{2} \times 5 = 7.5°$

Question 2. A‘ short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is
(a) blue (b) green (c) violet (d) red
Solution: (d) As velocity of wave is given by the relation v = f λ . When light ray goes from one medium to other medium, the frequency of light remains unchanged. Hence v ∝ λ or greater the wavelength, greater the speed.
The light of red colour is of highest wavelength and therefore of highest speed. Therefore, after travelling through the slab, the red colour emerges first.

Question 3. An object approaches a convergent lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image
(a) moves away from the lens with an uniform speed 5 m/s
(b) moves away from the lens with an uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration
Solution: (c)
In our problem the object approaches a convergent lens from the left of the lens with a uniform speed of 5 m/s, hence the image will move away from the lens with a non-uniform acceleration, the image moves slower in the beginning and faster later on will move from F to 2F and when the object moves from 2F to F,the image will move from 2F to infinity. At 2F, the speed of the object and image will be equal.

Question 4. A passenger in an aeroplane shall
(a) never see a rainbow
(b) may see a primary and a secondary rainbow as concentric circles
(c) may see a primary and a secondary rainbow as concentric arcs
(d) shall never see a secondary rainbow
Solution: (b) As aeroplane is at higher altitude, the passenger in an aeroplane may see a primary and a secondary rainbow like concentric circles.

Key concept:

- If an object move with constant speed (V_o) towards a convex lens from infinity to focus, the image will move slower in the beginning and then faster. Also $V_i = \left(\frac{f}{f+u}\right)^2 \cdot V_o$.
- If an object approaches the lens, the image moves away from lens with a non-uniform acceleration.

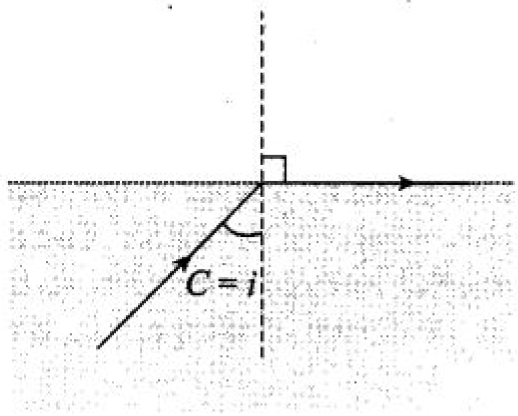
Question 5. You are given four sources of light each one providing a light of a single colour —red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90°. Which of the following statements is correct if the source of yellow light is replaced with that of other lights without changing the angle of incidence?
(a) The beam of red light would undergo total internal reflection.
(b) The beam of red light would bend towards the normal while it gets refracted through the second medium.
(c) The beam of blue light would undergo total internal reflection.
(d) The beam of green light would bend away from the normal as it gets refracted through the second medium.

Solution: (c)

Key concept: *According to Cauchy relationship,*

$$\lambda \propto \frac{1}{\mu}$$

Smaller the wavelength higher the refractive index and consequently smaller the critical angle.



We know $v = f\lambda$, the frequency of wave remains unchanged with medium hence $v \propto \lambda$.

The critical angle, $\sin C = \frac{1}{\mu}$

Also, velocity of light, $v \propto \frac{1}{\mu}$

According to VIBGYOR, among all given sources of light, the blue light have smallest wavelength. As $\lambda_{blue} < \lambda_{yellow}$ hence $v_{blue} < v_{yellow}$, it means $\mu_{blue} > \mu_{yellow}$.

It means critical angle for blue is less than yellow colour, the critical angle is least which facilitates total internal reflection for the beam of blue light.

Question 6. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will
(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies

Solution: (c)

Key concept: The relation between f, μ, R_1 and R_2 is known as lens maker's formula and it is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$R_1 = \infty, R_2 = -R$$

$$f = \frac{R}{(\mu - 1)}$$

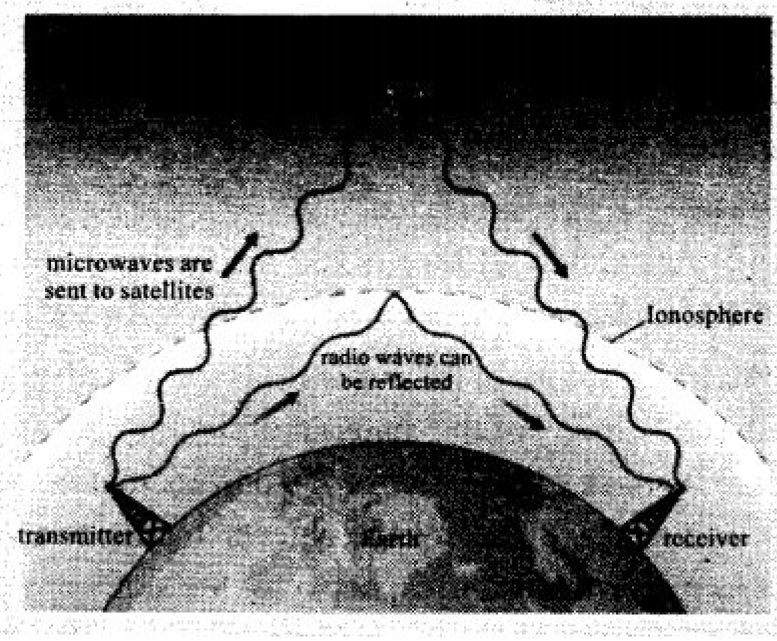
Here, $R = 20$ cm, $\mu = 1.5$. On substituting the values, we get

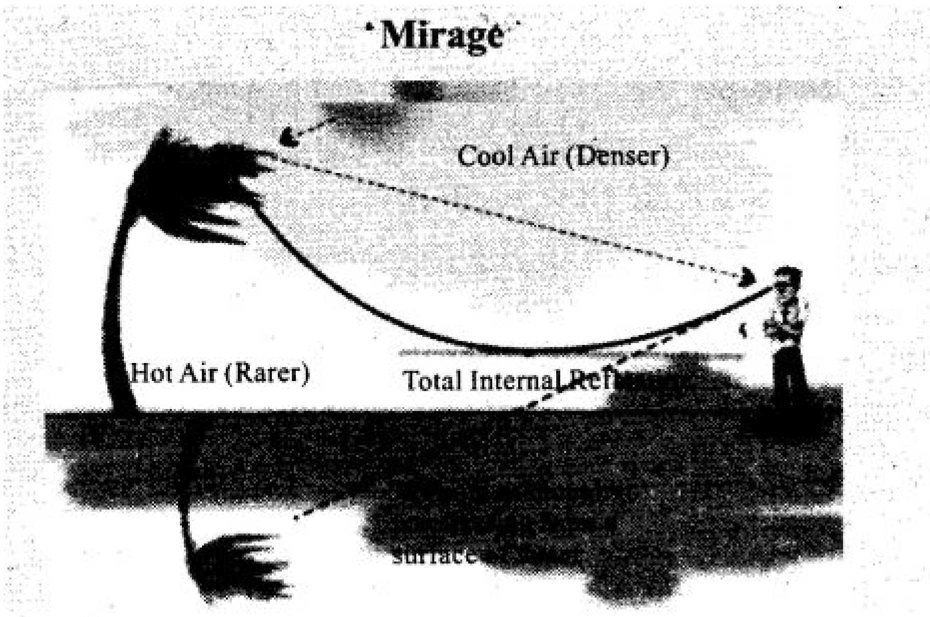
$$f = \frac{R}{\mu - 1} = \frac{20}{15 - 1} = 40 \text{ cm}$$

As $f > 0$ means converging nature. Therefore, lens act as a convex lens irrespective of the side on which the object lies.

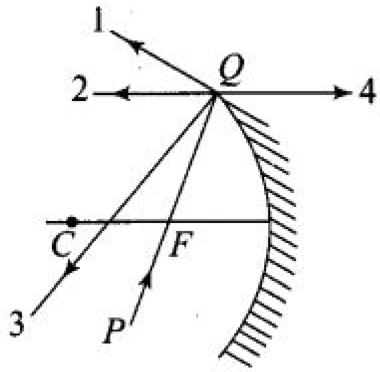
Question 7. The phenomena involved in the reflection of radio waves by ionosphere is similar to
(a) reflection of light by a plane mirror
(b) total internal reflection of light in air during a mirage
(c) dispersion of light by water molecules during the formation of a rainbow
(d) scattering of light by the particles of air
Solution: (b) Radio waves are reflected by a layer of atmosphere called the Ionosphere, so they can reach distant parts of the Earth. The reflection of radio waves by ionosphere is due to total internal reflection. It is the same as total internal reflection of light in air during a mirage, i.e., angle of incidence is greater than critical angle.
Important point: The ionized part of the Earth's atmosphere is known as the ionosphere. Ultraviolet light from the sun collides with atoms in this region knocking electrons loose. The creates ions, or atoms with missing electrons. This is what gives the Ionosphere its name- and it is the free electrons that cause the reflection and absorption of ratio waves.

Reflection of radiowaves by ionosphere



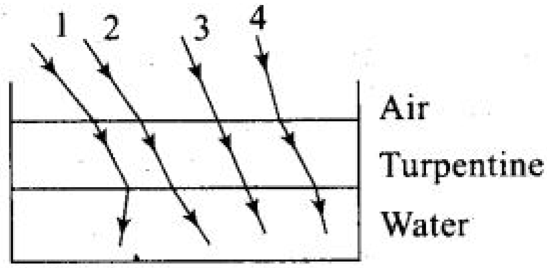


Question 8. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray?



(a) 1 (b) 2
(c) 3 (d) 4
Solution: (b) The ray PQ of light passes through focus F and incident on the concave mirror, after reflection, should become parallel to the principal axis and shown by ray 2 in the figure.
Important points:
We can locate the image of any extended object graphically by drawing any two of the following four special rays:
1. A ray initially parallel to the principal axis is reflected through the focus of the mirror (1).
2. A ray passing through the center of curvature is reflected back along itself (3).
3. A ray initially passing through the focus is reflected parallel to the principal axis (2).
4. A ray incident at the pole is reflected symmetrically.

Question 9. The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?

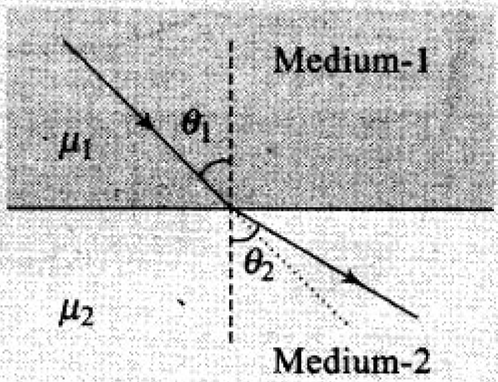


(a) 1 (b) 2 (c) 3 (d) 4
Solution: (b)

Key concept: The Snell's law describes the relation between angle of incidence θ_1 and angle of refraction θ_2:

$$\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2 = \text{constant} \quad \ldots(1)$$

where μ_1 and μ_2 are refractive indices of the two media.



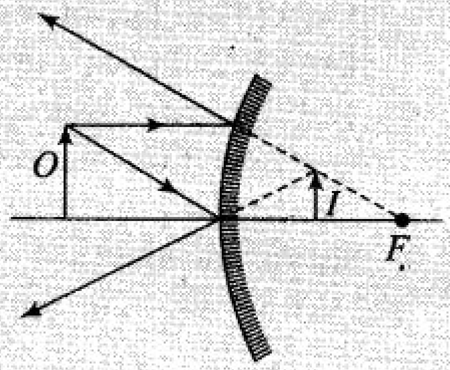
When a light ray goes from (optically) denser medium to (optically) rarer medium, then it bends away the normal, i.e., $\theta_1 < \theta_2$ and vice-versa.

Here, light ray goes from (optically) rarer medium air to optically denser medium turpentine, then it bends towards the normal, i.e., $\theta 1 > \theta 2$ whereas when it goes from to optically denser medium turpentine to rarer medium water, then it bends away the normal.

Question 10. A far is moving with a constant speed of 60 km h^{-1} on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of 100 m and is approaching with a speed of 5 km h^{-1}.
In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement (s) is/are correct?
(a) The speed of the car in the rear is 65 km h^{-1}
(b) In the side mirror, the car in the-rear would appear to approach with a speed of 5 km h-1 to the driver of the leading car
(c) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases
(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases
Solution: (d)

Key concept:

- *Object placed in front of mirror*: For all positions of object in front of a mirror, image is virtual, erect and smaller in size.
- As object moves towards the pole, magnification increases and tends to unity at pole.



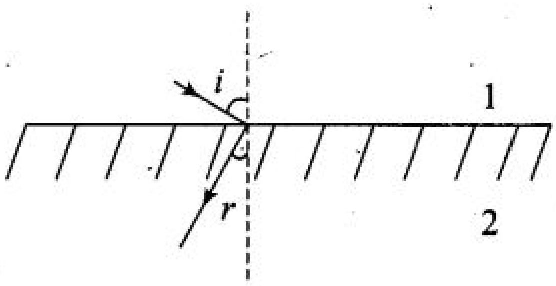
- *Object moving along the principal axis*: On differentiating the mirror formula with respect to time, we get $\frac{dv}{dt} = -\left(\frac{v}{u}\right)^2 \frac{du}{dt} = -\left(\frac{f}{u-f}\right)^2 \cdot \frac{du}{dt}$

  where dv/dt is the velocity of image along the principal axis and du/dt is the velocity of object along the principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of the object.

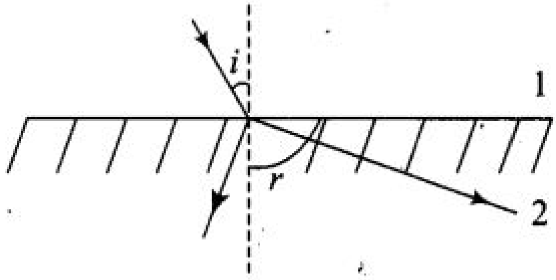
As the distance between the cars decreases, the speed of the image of the car would appear to increase.

Question 11. There are certain material developed in laboratories which have a negative refractive index figure. A ray incident from air (Medium 1) into such a medium (Medium 2) shall follow a path given by

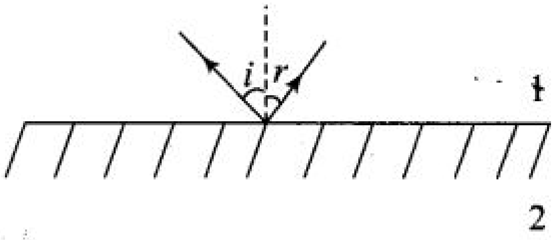
(a)



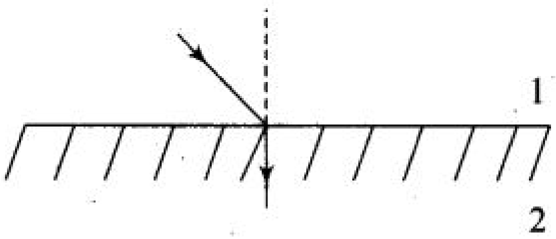
(b)



(c)



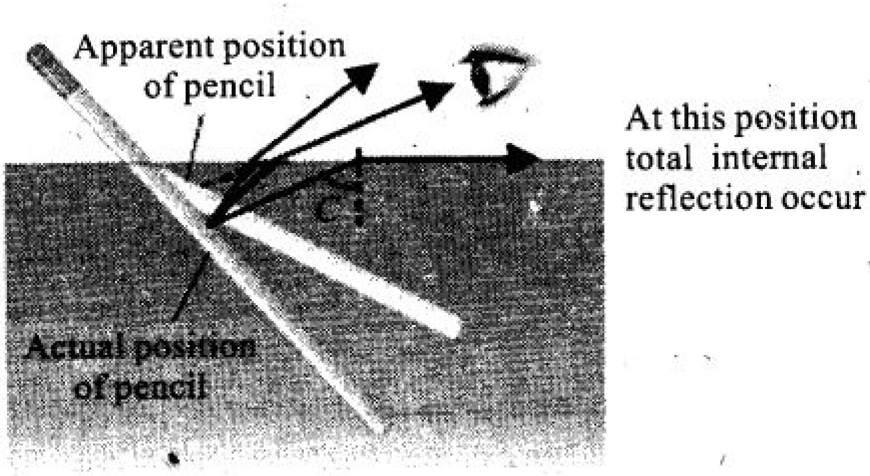
(d)



Solution: (a) The materials with negative refractive index responds to Snell's law just opposite way. If incident ray from air (Medium 1) incident on those material, the ray refract or bend same side of the normal as in option (a).

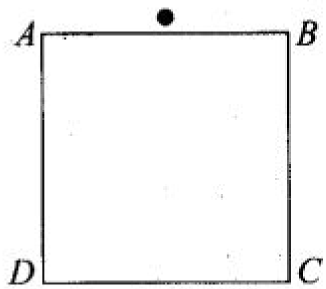
One or More Than One Correct Answer Type

Question 12. Consider an extended object immersed in water contained in a plane trough. When seen from close to the edge of the trough the object looks distorted because
(a) the apparent depth of the points close to the edge are nearer the surface of the water compared to the points away from the edge
(b) the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air
(c) some of the points of the object far away from the edge may not be visible because of total internal reflection
(d) water in a trough acts as a lens and magnifies the object
Solution: (a, b, c)
Key concept: The light from the pencil is refracted when it passes from the water into air, bending away from the normal as it moves from high to low refractive index.

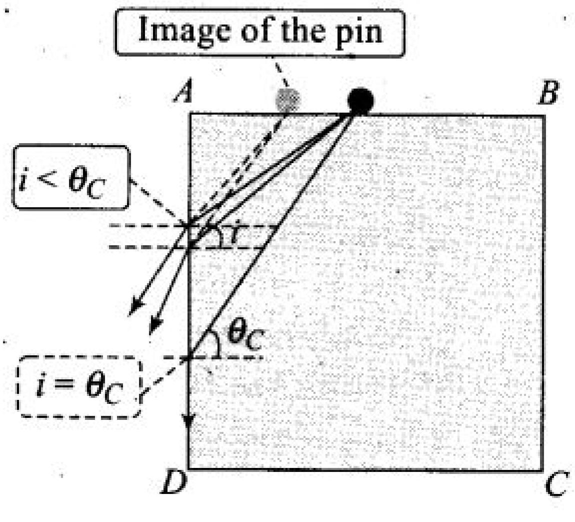


When light from the submerged object before reaching to the observer gets, refracted from water surface, the rays bend away from normal and the angle subtended by the image of the object at the eye is smaller than the actual angle subtended by the object in air. Also the apparent depth of the .points close to the edge are nearer the surface of the water compared to the points away from the edge.
As we move towards right, the angle of incident increases and becomes equal to critical angle. Hence some of the points of the object far away from the edge may not be visible because of total internal reflection.

Question 13. A rectangular block of glass ABCD has a refractive index 1.6. A pin is placed midway on the face AB of figure. When observed from the face AD, the pin shall



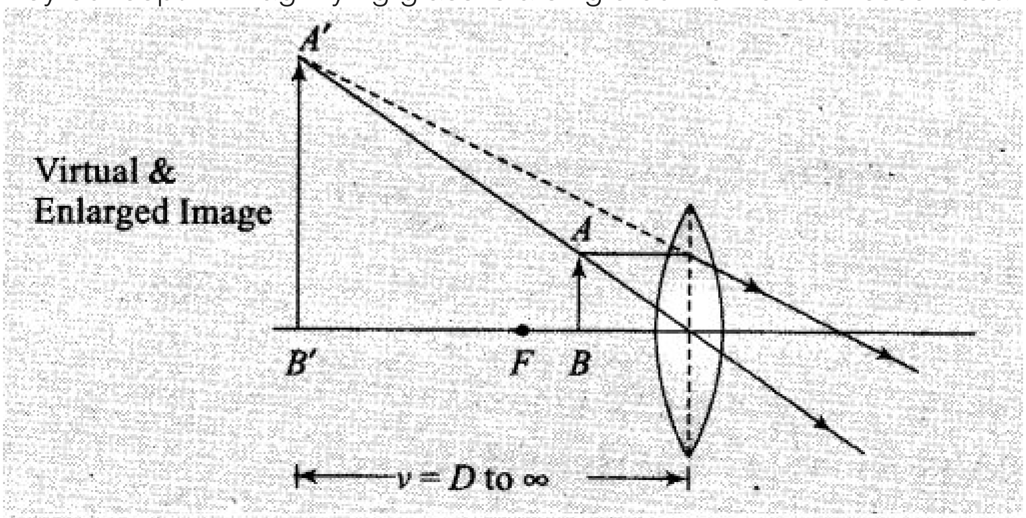
(a) appear to be near A
(b) appear to be near D
(c) appear to be at the centre of AD
(d) not be seen at all
Solution: (a, d) As long as angle of incidence on AD of the ray emanating from pin is less than the critical angle, the pin shall appear to be near A.



When angle of incidence on AD of the ray emanating from pins is greater than the critical angle, the light suffers from total internal reflection and cannot be seen through AD.

Question 14. Between the primary and secondary rainbow, these is a dark band known as Alexandar's dark band. This is because
(a) light scattered into this region interfere destructively
(b) there is no light scattered into this region
(c) light is absorbed in this region
(d)angle made at the eye by the scattered rays with respect to the incident light of the sun lies between approximately 42° and 50°
Solution: (a, d) The Alexandar's dark band lies between the primary and secondary rainbows, formed due to light scattered into this region interfere destructively. The primary rainbows subtends an angle nearly 41° to 42° at observer's eye, whereas secondary rainbows subtends an angle nearly 51° to 54° at observer's eye w.r.t. incident light ray.
Hence, the scattered rays with respect to the incident light of the sun lies between approximately 42° and 50°.

Question 15. A magnifying glass is used, as the object to be viewed can be brought closer to the eye than the normal near point. This results in
(a) a larger angle to be subtended by the object at the eye and hence, viewed in greater detail
(b) the formation of a virtual erect image
(c) increase in the field of view
(d) infinite magnification at the near point
Solution: (a, b)
Key concept: A magnifying glass is a single convex lens of lesser focal length.



For magnification when final image is formed at D and ∞ (*i.e.*, m_D and m_∞).

$$m_D = \left(1 + \frac{D}{f}\right)_{max} \quad \text{and} \quad m_\infty = \left(\frac{D}{f}\right)_{min}$$

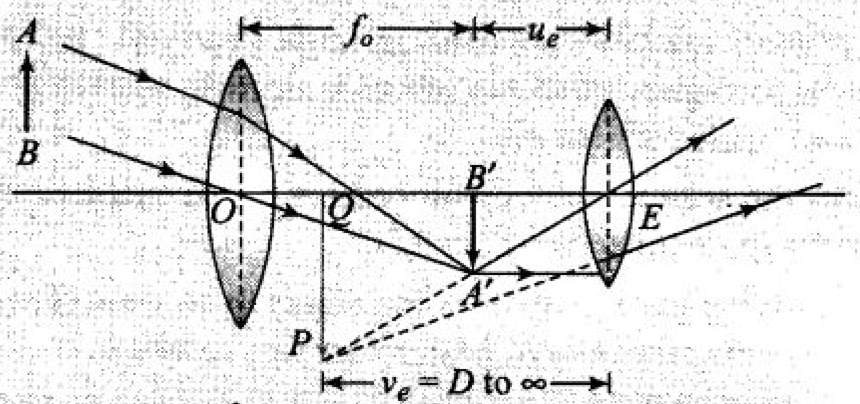
When a magnifying glass is used, the object to be viewed can be brought closer to the eye than the normal near point. This results in a larger angle to be subtended by the object at the eye and hence, viewed in greater detail. Moreover, the formation of a virtual erect and enlarged image takes place.

Question 16. An astronomical refractive telescope has an objective of focal length 20 m and an eyepiece of focal length 2 cm.
(a) The length of the telescope tube is 20.02 m
(b) The magnification is 1000
(c) The image formed is inverted

(d) An objective of a larger aperture will increase the brightness and reduce chromatic aberration of the image
Solution: (a, b, c)

Key concept:

Astronomical telescope



- Used to see heavenly bodies.
- $f_{objective} > f_{eye\ lens}$ and $d_{objective} > d_{eye\ lens}$
- Intermediate image is real, inverted and small.
- Final image is virtual, inverted and small.
- **Magnification :** $m_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

  and $m_\infty = -\frac{f_0}{f_e}$
- **Length** : $L_D = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$

  and $L_\infty = f_0 + f_e$

The length of the telescope tube is $f_0 + f_e = 20 + (0.02) = 20.02$ m

Also, $m = 20/0.02 = 1000$

Also, the image formed is inverted.

Very Short Answer Type Questions

Question 17. Will the focal length of a lens for red light be more, same or less than that for blue light?

Solution:

Key concept: The refractive index depends on colour of light or wavelength of light.

Cauchy's equation: $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \dots$

As $\lambda_{red} > \lambda_{blue}$ hence $\mu_{red} < \mu_{blue}$

Hence parallel beams of light incident on a lens will be bent more towards the axis for blue light compared to red.

By lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The refractive index for red light is less than that for blue light, $\mu_{red} < \mu_{blue}$

Hence $\frac{1}{f_{red}} < \frac{1}{f_{blue}} \Rightarrow f_{red} > f_{blue}$

Thus, the focal length for red light will be greater than that for blue light.

Question 18. The near vision of an average person is 25 cm. To view an object with an angular magnification of 10, what should be the power of the microscope?
Solution:

It is given, the least distance of distinct vision of an average person (i.e., D) is 25 cm, in order to view an object with magnification 10,

Here, $v = D = 25$ cm and $u = f$

But the magnification $m = \frac{v}{u} = \frac{D}{f}$

$$\Rightarrow \quad f = \frac{D}{m} = \frac{25}{10} = 2.5 \text{ cm} = 0.025 \text{ m}$$

But power $P = \frac{1}{f(\text{in m})} = \frac{1}{0.025} = 40$ D

This is the required power of lens.

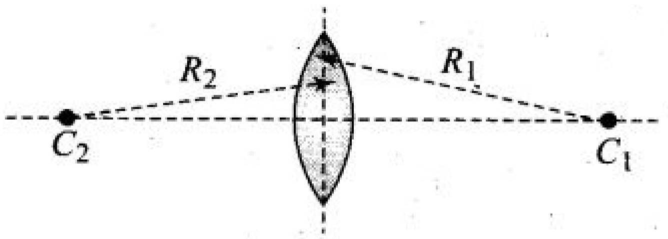
Question 19. An unsymmetrical double convex thin lens forms the image of a point object on its axis. Will the position of the image change if the lens is reversed?
Solution:

Key concept: Thin lens formula: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

For a given object position if focal length of the lens does not change, the image position remains unchanged.

By lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



For this position R_1 is positive and R_2 is negative. Hence focal length at this position

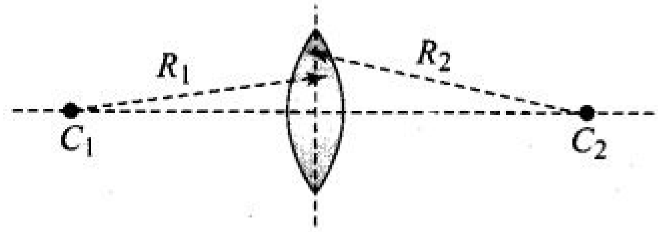
$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{(+R_1)} - \frac{1}{(-R_2)}\right) = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Now the lens is reversed,

At this position, R_2 is positive and R_1 is negative. Hence focal length at this position is

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{(+R_2)} - \frac{1}{(-R_1)}\right) = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

We can observe the focal length of the lens does not change in both positions, hence the image position remains unchanged.



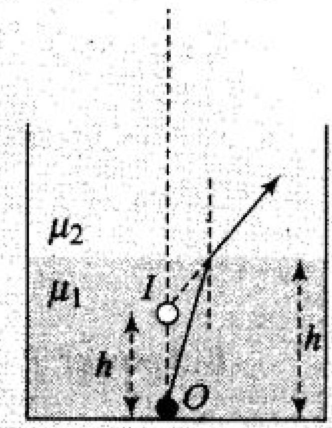
Question 20. Three immiscible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is h/3. A dot is made at the bottom of the beaker. For near normal vision, find the apparent depth of the dot.

Solution:

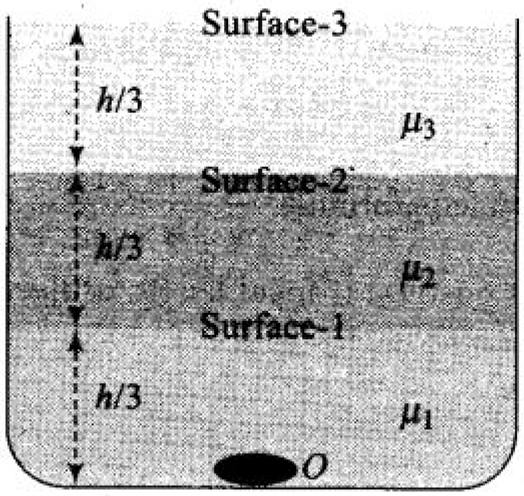
Key concept: *Coordinate convention*: At the first surface (+ upward and −ve downward)

$\frac{\mu_2}{h'} - \frac{\mu_1}{(-h)} = \frac{\mu_2 - \mu_1}{\infty}$ (infinity because the surface is plane),

or $h' = -\frac{\mu_2}{\mu_1}h$. The negative sign shows that it is on the side of the object



h' is the apparent depth of O after refraction from interface.



The position of image of O after refraction from surface-1. If seen from μ_2, the apparent depth is h_1

$$h_1 = -\frac{\mu_2}{\mu_1}\frac{h}{3}$$

The negative sign shows that it is on the side of the object.

Since, the image formed by surface-1. will act as an object for surface-2. If seen from μ_3, the apparent depth is h_2 .

Similarly, the image formed by Medium 2, O_2 acts as an object for Medium 3.

$$h_2 = -\frac{\mu_3}{\mu_2}\left(\frac{\mu_2}{\mu_1}\frac{h}{3} + \frac{h}{3}\right) = -\frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_2}{\mu_1}\right)$$

Finally the image formed by surface-2 will act as an object for surface- 2. If seen from outside, the apparent depth is h_3.

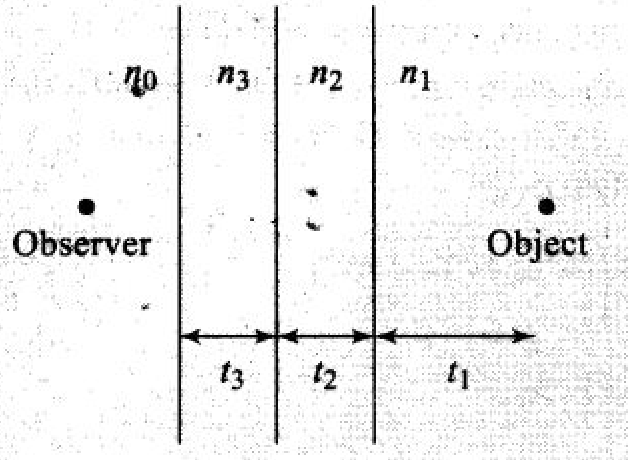
$$h_3 = -\frac{1}{\mu_3}\left[\frac{h}{3} + \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right] = -\frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)$$

Hence apparent depth of dot is $\frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)$

Important point:

Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1\text{real}}} + \frac{t_2}{n_{2\text{real}}} + \frac{t_3}{n_{3\text{real}}} + \ldots + \frac{t_n}{n_{n\text{real}}}$$



Question 21. For a glass prism (μ= √3), the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism.

Solution:

The refractive index of prism angle A and angle of minimum deviation δ_m is given by

$$\mu = \frac{\sin\left[\frac{(A+\delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

Here we are given, $\delta_m = A$

Substituting the value, we have $\mu = \dfrac{\sin A}{\sin \frac{A}{2}}$

$$\Rightarrow \quad \mu = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

$$\Rightarrow \quad \mu = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

For the given value of refractive index,

we have, $\cos\frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{A}{2} = 30^\circ$

or $\quad A = 60^\circ$

This is the required value of prism angle.

Short Answer Type Questions

Question 22. A short object of length L is placed along the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f what will be the length of the image? You may take L << Iv – f I.

Solution:

Thin mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

u = object distance and v = image distance.

Since, the object distance is u. Let us consider the two ends of the object be at distance u_1 and u_2 respectively, so that $du = |u_1 - u_2| = L$. Hence size of image can be written as dv.

By differentiating both sides $\left(-\frac{dv}{v^2}\right)+\left(-\frac{du}{u^2}\right)=0 \Rightarrow \frac{dv}{v^2}=-\frac{du}{u^2}$

or $\quad dv=-\left(\frac{v^2}{u^2}\right)du \qquad$ ...(i)

As $\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \Rightarrow v=\frac{fu}{u-f} \Rightarrow v=\frac{fu}{u-f}$

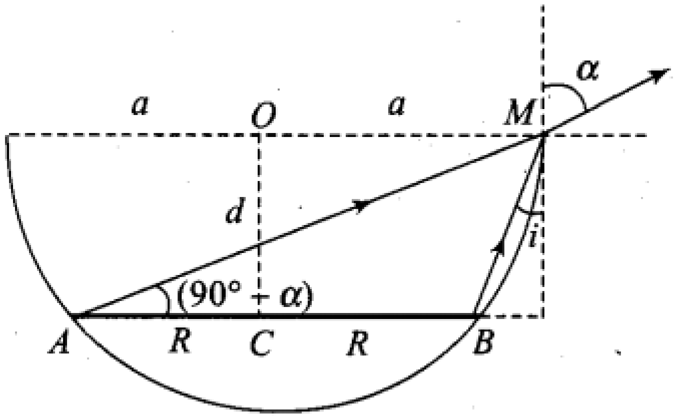
Or $\quad \frac{v}{u}=\frac{f}{u-f} \qquad$ ...(ii)

From (i) and (ii) $dv=-\left(\frac{f}{u-f}\right)^2 du$

But $du = L$, hence the length of the image is $\frac{f^2}{(u-f)^2}L$

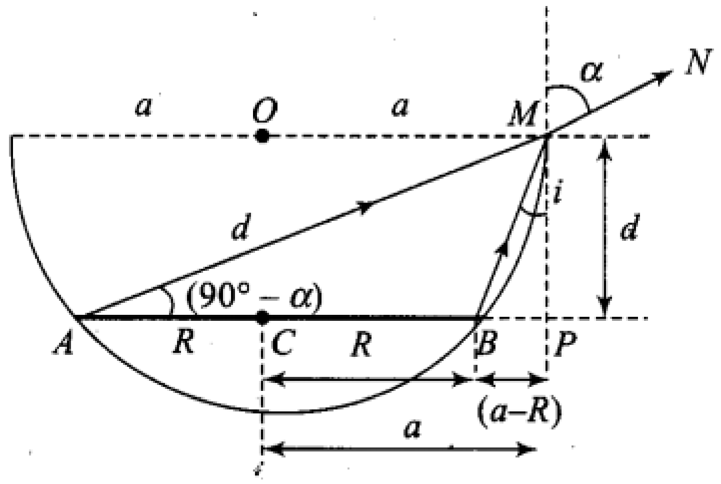
This is the required expression for length of image.

Question 23. A circular disc of radius R is placed co-axially and horizontally inside an opaque hemispherical bowl of radius a (figure). The far edge of the disc is just visible when viewed from the edge of the bowl. The bowl is filled with transparent liquid of refractive index μ. and the near edge of the disc becomes just visible. How far below the top of the bowl is the disc placed?



Solution:

Referring to the figure, AM is the direction of incidence ray before filling the liquid. After filling the liquid in bowl, BM is the direction of the incident ray. Refracted ray in both cases is same as that along MN.



Let the disc is separated by O at a distance d as shown in figure. Also, considering angle

$$P = 90°, OM = a, CB = R, BP = a - R, AP = a + R$$

Here, in ΔBMP, $\sin i = \frac{BP}{BM} = \frac{a-R}{\sqrt{d^2+(a-R)^2}}$ ...(i)

and in ΔAMP $\cos(90° - \alpha) = \sin\alpha = \frac{a+R}{\sqrt{d^2+(a+R)^2}}$ ...(ii)

But on applying Snell's law at point M

$$\mu \times \sin i = 1 \times \sin r$$

$$\mu \times \frac{a-R}{\sqrt{d^2+(a-R)^2}} = 1 \times \frac{a+R}{\sqrt{d^2+(a+R)^2}}$$

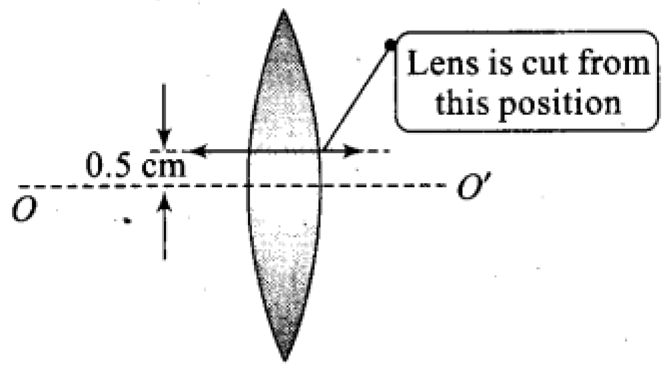
$$\Rightarrow \quad d = \frac{\mu(a^2-b^2)}{\sqrt{(a+r)^2-\mu(a-r)^2}}$$

Question 24. thin convex lens of focal length 25 cm is cut into two pieces 0.5 cm above the principal axis. The top part is placed at (0, 0) and an object is placed at (-50 cm, 0). Find the coordinates of the image.

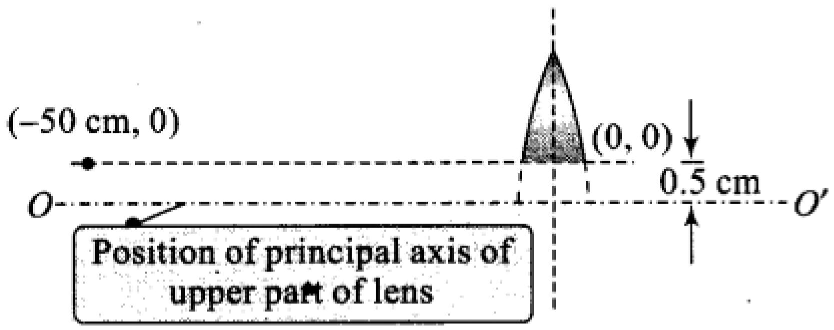
Solution:

Key concept: If a symmetric lens is cut parallel to principal axis in two parts. Focal length remains the same for each part. Intensity of image formed by each part will be less compared as that of complete lens.

If there was no cut, then the object would have been at a height of 0.5 cm from the principal axis OO'.



The top part is placed at (0, 0) and an object placed at (–50 cm, 0). There is no effect on the focal length of the lens.



Applying lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-50} + \frac{1}{25} = \frac{1}{50} \Rightarrow v = 50 \text{ cm}$$

Magnification is $m = \frac{v}{u} = -\frac{50}{50} = -1$

Hence the image would have been formed at 50 cm from the pole and 0.5 cm below the principal axis. Hence, with respect to the X-axis passing through the edge of the cut lens, the coordinates of the image are (50 cm, –1 cm).

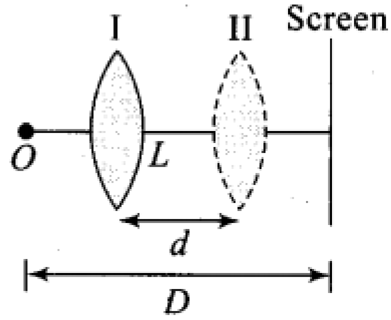
Question 25. In many experimental set-ups, the source and screen are fixed at a distance say D and the lens is movable. Show that there are two positions for the lens for which an image is formed on the screen. Find the distance between these points and the ratio of the image sizes for these two points.

Solution:

Key Concept: This is also one of the methods for finding focal length of the length in laboratory and knows as “Displacement method”.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Consider a convex lens L placed between an object O and a screen S. The distance between the object and the screen is D and the positions of the object and the screen are held fixed. The lens can be moved along the axis of the system and at a position I a sharp image will be formed on the screen. Interestingly, there is another position on the same axis where a sharp image will once again be obtained on the screen. The position is marked as II in Figure.



In the figure, let the distance of position I from the object be x_1. Then the distance of the screen from the lens is $D - x_1$.

Therefore, $u = -x_1$ and $v = +(D - x_1)$.

Placing it in the lens formula,

$$\frac{1}{D - x_1} - \frac{1}{(-x_1)} = \frac{1}{f} \qquad \text{...(i)}$$

At position II, let the distance of the lens from the screen be x_2. Then the distance of the screen from the lens is $D - x_2$.

Therefore, $u = -x_2$ and $v = +(D - x_2)$.

Placing it in the lens formula

$$\frac{1}{D - x_2} - \frac{1}{(-x_2)} = \frac{1}{f} \qquad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we realize that there are only two solutions:

1. $x_1 = x_2$; or
2. $D - x_1 = x_2$ and $D - x_2 = x_1$

The first solution is trivial. Therefore, if the first position of the lens, for a sharp image, is x_1 from the object, the second position is at $D - x_1$ from the object.

Let the distance between the two positions I and II be d.

From the diagram, it is clear that

$$D = x_1 + x_2 \text{ and } d = x_2 - x_1 \qquad \text{...(iii)}$$

On solving, two equations in (iii) we have

$$x_1 = \frac{D - d}{2} \text{ and } D - x_1 = \frac{D + d}{2} \qquad \text{...(iv)}$$

Substituting Eq. (iv) in Eq. (i), we get

$$\frac{1}{\left(\frac{D+d}{2}\right)}-\frac{1}{\left(-\frac{D-d}{2}\right)}=\frac{1}{f} \Rightarrow f=\frac{D^2-d^2}{4D} \qquad \ldots(v)$$

or $\quad d=\sqrt{D^2-4Df} \qquad \ldots(vi)$

If $u=\frac{D}{2}+\frac{d}{2}$, then the image is at $v=\frac{D}{2}-\frac{d}{2}$.

$\therefore$ The magnification $m_1=\frac{D-d}{D+d}$

If $u=\frac{D-d}{2}$, then $v=\frac{D+d}{2}$

$\therefore$ The magnification $m_2=\frac{D+d}{D-d}$

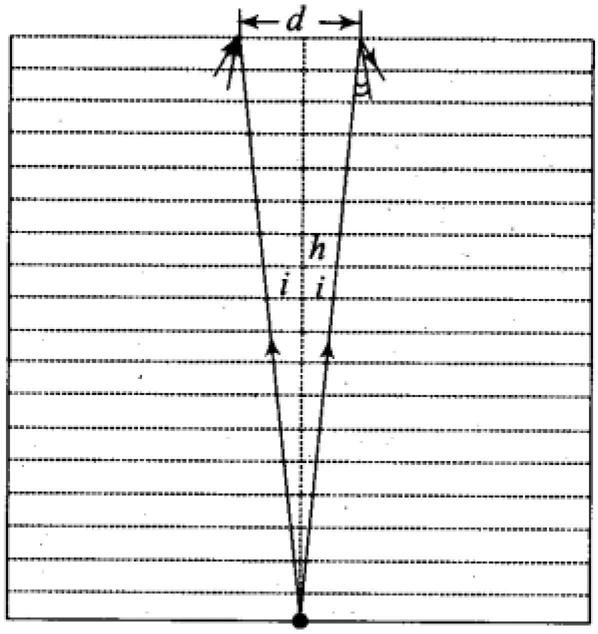
Thus, $\quad \frac{m_2}{m_1}=\left(\frac{D+d}{D-d}\right)^2$.

This is the required expression of magnification.

Important points:

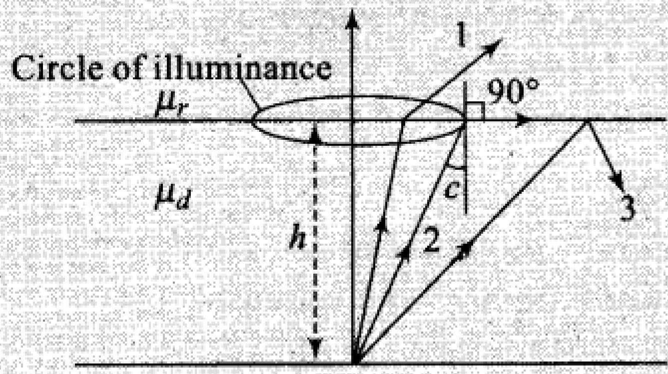
- We notice from Eq. (vi) that a solution for d is possible only when $D \geq 4f$.
- When $D < 4f$, there is no position for which a sharp image can be formed.
- When $D = 4f$, there is only one position where a sharp image is formed.
- When $D > 4f$, there are two positions where a sharp image is formed.
- The method is suitable for convex lenses only.

Question 26. A jar of height h is filled with a transparent liquid of refraction index μ (figure).At the center of the jar on the bottom surface is a dot.Find the minimum diameter of a disc,such that when placed on the top surface symmetrically about the center,the dot is invisible.



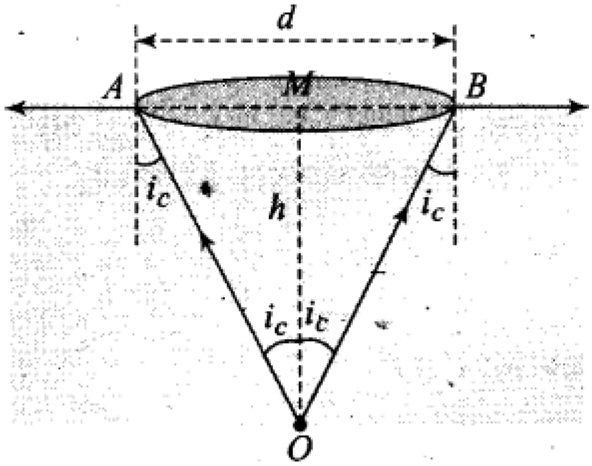
Solution:

Key concept:



In the figure, ray 1 strikes the surface at an angle less than critical angle c and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called ***circle of illuminance*** (C.O.I). All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in the rarer medium and then the object cannot be seen from the rarer medium.

In figure, O is a small dot at the bottom of the jar. The ray from the dot emerges out of a circular patch of water surface of diameter AB till the angle of incidence for the rays OA and OB exceeds the critical angle (i_C).



Rays of light incident at an angle greater than i_C, are totally reflected within water and consequently cannot emerge out of the water surface.

As $\sin i_C = \frac{1}{\mu} \Rightarrow \tan i_C = \frac{1}{\sqrt{\mu^2 - 1}}$,

Now, $\frac{d/2}{h} = \tan i_c$

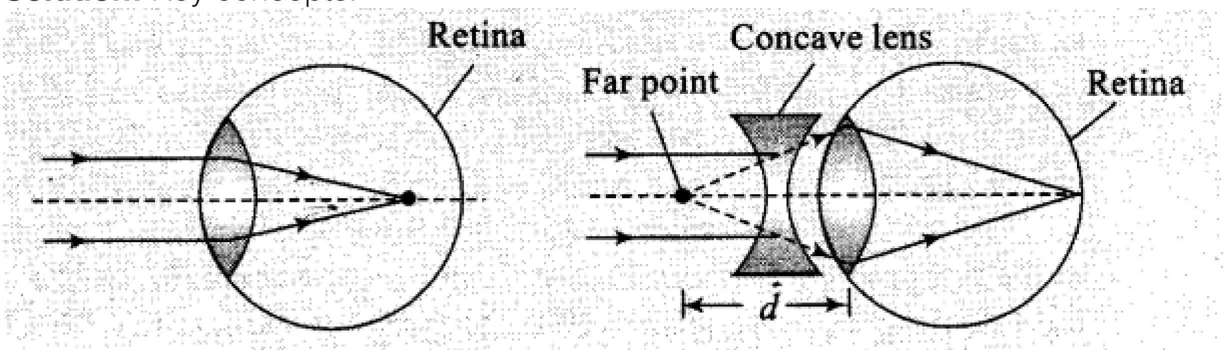
$\Rightarrow \frac{d}{2} = h \tan i_c = h \frac{1}{\sqrt{\mu^2 - 1}}$

$\Rightarrow d = \frac{2h}{\sqrt{\mu^2 - 1}}$

This is the required expression of d.

Long Answer Type Questions

Question 27. A myopic adult has a far point at 0.1 m. His power of accommodation is 4 D.
(i) What power lenses are required to see distant objects?
(ii) What is his near point without glasses?
(iii) What is his near point with glasses? (Take the image distance from the lens of the eye to the retina to be 2 cm.)
Solution: Key concepts:



(a) Defected eye (b) Removal of Defect

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

in terms of power $P = P_1 + P_2$

(i) If for the normal relaxed eye of an average person, the power at the far point be P_f. The required power

$$P_f = \frac{1}{f} = \frac{1}{0.1} + \frac{1}{0.02} = 60 \text{ D}$$

By the corrective lens the object distance at the far point is ∞.

The power required is, $P_f' = \frac{1}{f'} = \frac{1}{\infty} + \frac{1}{0.02} = 50 \text{ D}$

Now for eye + lens system, we have the sum of the eye and that of the glasses P_g

$$P'_f = P_f + P_g \Rightarrow 50\text{D} = 60\text{D} + P_g$$

which gives, $P_g = -10\text{D}$

(ii) For the normal eye his power of accommodation is 4D. Let the power of the normal eye for near vision be P_n.

Then, $4 = P_n - P_f$ or $P_n = 64\text{D}$

Let his near point be x_n, then

$$\frac{1}{x_n} + \frac{1}{0.02} = 64 \text{ or } \frac{1}{x_n} + 50 = 64$$

$$\frac{1}{x_n} = 14 \Rightarrow x_n = \frac{1}{14} \text{ m} = 0.07 \text{ m}$$

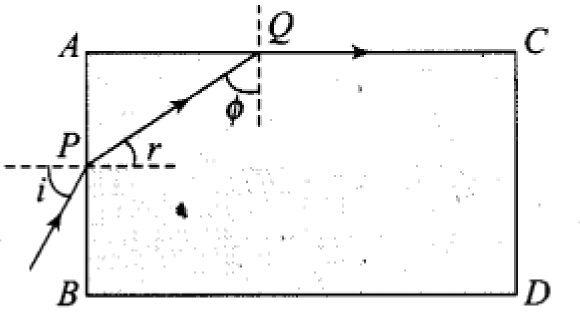
(iii) With glasses $P_n' = P_f' + 4 = 54$

$$54 = \frac{1}{x_n'} + \frac{1}{0.02} = \frac{1}{x_n'} + 50$$

$$\frac{1}{x_n'} = 4 \Rightarrow x_n' = \frac{1}{4} \text{ m} = 0.25 \text{ m}$$

Question 28. Show that for a material with refractive index μ ≥ √2, light incident at angle shall be guided along, a length perpendicular to the incident face.

Solution: Let the ray incident on face AB at angle i, after refraction, it travels along PQ and then interact with face AC which is perpendicular to the incident face.



Any ray is guided along AC if the angle ray makes with the face AC (ϕ) is greater than the critical angle. From figure

$\phi + r = 90°$, therefore $\sin\phi = \cos r$ ...(i)

If ϕ is the critical angle it means, $\sin\phi \geq \frac{1}{\mu}$ ...(ii)

From (i) and (ii), $\cos r \geq \frac{1}{\mu^2}$ or $1 - \cos^2 r \leq 1 - \frac{1}{\mu^2}$

i.e., $\sin^2 r \leq \frac{1}{\mu^2} \Rightarrow \sin^2 r \leq 1 - \frac{1}{\mu^2}$ ...(iii)

Applying Snell's law on face AB,

$1 \cdot \sin i = \mu \sin r$

or $\sin^2 i = \mu^2 \sin^2 r \Rightarrow \sin^2 r = \frac{1}{\mu^2}\sin^2 i$ ...(iv)

From (i) and (ii), $\frac{1}{\mu^2}\sin^2 i \leq 1 - \frac{1}{\mu^2}$

or $\sin^2 i \leq \mu^2 - 1$ ...(v)

When $i = \frac{\pi}{2}$, then we have smallest angle ϕ.

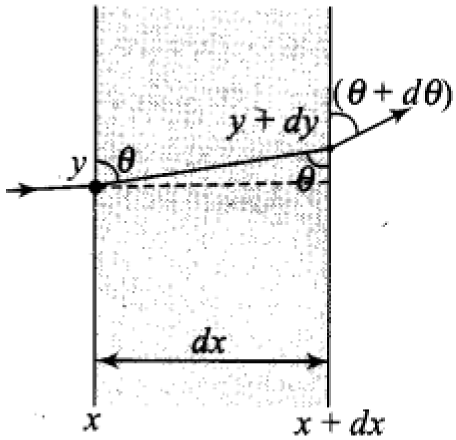
If it is greater than the critical angle, then all other angles of incidence shall be more than the critical angle.

Thus, $1 \leq \mu^2 - 1$ or $\mu^2 \geq 2$

$\Rightarrow \mu \geq \sqrt{2}$. This is the required result.

Question 29. The mixture of a pure liquid and a solution in a long vertical column (i.e., horizontal dimensions << vertical dimensions) produces diffusion of solute particles and hence a refractive index gradient along the vertical dimension. A ray of light entering the column at right angles to the vertical is deviated from its original path. Find the deviation in travelling a horizontal distance d << h, the height of the column.
Solution:

Let us consider a portion of a ray between x and $x + dx$ inside the liquid solution. Let the angle of incidence of ray at x be θ and let the ray enters the thin column at height y. Because of the refraction it deviates from the original path and emerges at $x + dx$ with an angle $\theta + d\theta$ and at a height $y + dy$.



From Snell's law,

$$\mu(y) \sin \theta = \mu(y + dy) \sin (\theta + d\theta) \quad \ldots(i)$$

Let refractive index of the liquid at position y be $\mu(y) = \mu$, then

$$\mu(y + dy) = \mu + \left(\frac{d\mu}{dy}\right) dy = \mu + k dy$$

where $k = \left(\frac{d\mu}{dy}\right)$ = refractive index gradient along the vertical dimension.

Hence from (i), $\mu \sin \theta = (\mu + kdy) \cdot \sin (\theta + d\theta)$

$$\mu \sin \theta = (\mu + kdy) \cdot (\sin \theta \cdot \cos d\theta + \cos \theta \cdot \sin d\theta)$$

$$\mu \sin \theta = (\mu + kdy) \cdot (\sin \theta \cdot 1 + \cos \theta \cdot d\theta) \quad \ldots(ii)$$

For small angle $\sin d\theta \approx d\theta$ and $\cos d\theta \approx 1$

$$\mu \sin \theta = \mu \sin \theta + kdy \sin \theta + \mu \cos \theta \cdot d\theta + k \cos \theta dy \cdot d\theta$$

$$kdy \sin \theta + \mu \cos \theta \cdot d\theta = 0 \Rightarrow d\theta = -\frac{k}{\mu} \tan \theta \, dy$$

But $\tan \theta = \frac{dx}{dy}$ and $k = \left(\frac{d\mu}{dy}\right)$

$$d\theta = -\frac{k}{\mu}\left(\frac{dx}{dy}\right) dy \Rightarrow d\theta = -\frac{k}{\mu} dx$$

Integrating both sides, $\int_0^{\delta} d\theta = -\frac{k}{\mu}\int_0^{d} dx$

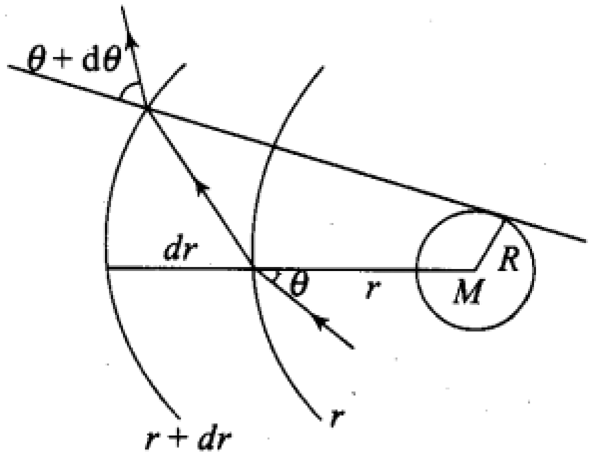
$$\Rightarrow \quad \delta = -\frac{kd}{\mu} = -\frac{d}{\mu}\left(\frac{d\mu}{dy}\right)$$

Question 30.

If light passes near a massive object, the gravitational interaction causes a bending of the ray. This can be thought of as happening due to a change in the effective refractive index of the medium given by

$$n(r) = 1 + 2\,GM/rc^2$$

where r is the distance of the point of consideration from the centre of the mass of the massive body, G is the universal gravitational constant, M the mass of the body and c the speed of light in vacuum. Considering a spherical object find the deviation of the ray from the original path as it grazes the object.



Solution:

Sol. Let us consider two spherical surfaces of radius r and $r + dr$. Let the light be incident at an angle θ at the surface at r and leave $r + dr$ at an angle $\theta + d\theta$. Then from Snell's law,

$$n(r)\sin\theta = n(r + dr)\sin(\theta + d\theta)$$

$$= \left(n(r) + \left(\frac{dn}{dr}\right)dr\right)(\sin\theta\cdot\cos d\theta + \cos\theta\cdot\sin d\theta)$$

$$\Rightarrow \quad n(r)\sin\theta = \left(n(r) + \left(\frac{dn}{dr}\right)dr\right)(\sin\theta + \cos\theta\cdot d\theta)$$

For small angle, $\sin d\theta \approx d\theta$ and $\cos d\theta \approx 1$

Ignoring the product of differentials

$$\Rightarrow \quad n(r)\sin\theta = n(r)\cdot\sin\theta + \left(\frac{dn}{dr}\right)dr\cdot\sin\theta + n(r)\cdot\cos\theta\cdot d\theta$$

or we have, $-\dfrac{dn}{dr}\tan\theta = n(r)\dfrac{d\theta}{dr}$

$$\frac{2GM}{r^2c^2}\tan\theta = \left(1 + \frac{2GM}{rc^2}\right)\frac{d\theta}{dr} \approx \frac{d\theta}{dr}$$

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{\tan\theta\, dr}{r^2}$$

Now, $r^2 = x^2 + R^2$ and $\tan\theta = \dfrac{R}{x}$

$$2rdr = 2xdx$$

Now substitution for integrals, we have

$$\int_0^{\theta_0} d\theta = \frac{2GM}{c^2}\int_{-\infty}^{\infty}\frac{R}{x}\frac{xdx}{(x^2 + R^2)^{\frac{3}{2}}}$$

Put $x = R\tan\phi$

$dx = R\sec^2\phi\, d\phi$

$$\therefore \quad \theta_0 = \frac{2GMR}{c^2}\int_{-\pi/2}^{\pi/2}\frac{R\sec^2\phi d\phi}{R^3\sec^3\phi}$$

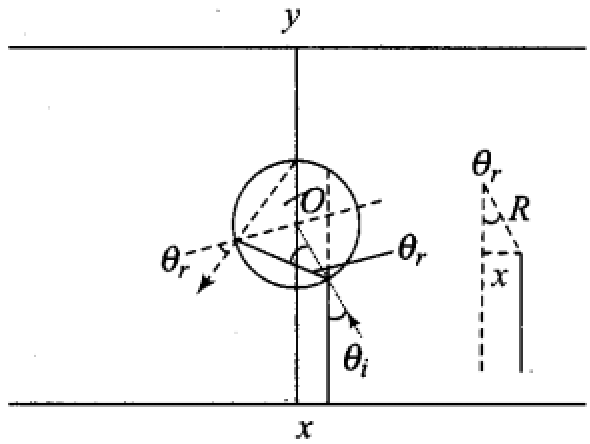
$$\theta_0 = \frac{2GM}{Rc^2}\int_{-\pi/2}^{\pi/2}\cos\phi d\phi = \frac{4GM}{Rc^2}$$

$\Rightarrow \theta_0 = \dfrac{4GM}{Rc^2}$. This is the required proof.

Question 31. An infinitely long cylinder of radius R is made of an unusual exotic material with refractive index -1 (figure). The cylinder is placed between two planes whose normals are along the y-direction. The centre of the cylinder O lies along they-axis.

A narrow laser beam is directed along the y-direction from the lower plate. The laser source is at a horizontal distance x from the diameter in the y-direction. Find the range of x such that light emitted from the lower plane does not reach the upper plane.

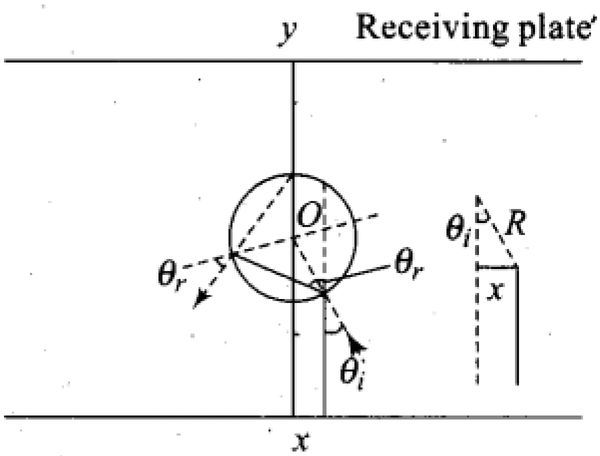
Solution:



Here the material has refractive index -1, θ_r is negative and θ'_r positive.

Now, $|\theta_i| = |\theta_r| = |\theta'_r|$

The total deviation of the outcoming ray from the incoming ray is $4\theta_r$. Rays shall not reach the receiving plate if $\frac{\pi}{2} \leq 4\theta_i \leq \frac{3\pi}{2}$ [angles measured clockwise from the y-axis]



On solving, $\frac{\pi}{8} \leq \theta_i \leq \frac{3\pi}{8}$

Now, $\sin \theta_i = \frac{x}{R}$

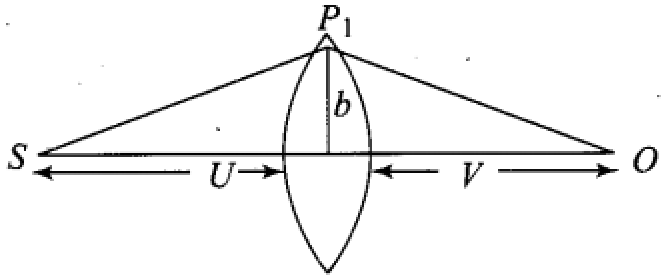
$$\frac{\pi}{8} \leq \sin^{-1} \frac{x}{R} \leq \frac{3\pi}{8}$$

or $\frac{\pi}{8} \leq \frac{x}{R} \leq \frac{3\pi}{8}$

Thus, for light emitted from the source shall not reach the receiving plate.

If $\frac{R\pi}{8} \leq x \leq \frac{R3\pi}{8}$.

Question 32. (i) Consider a thin lens placed between a source (S) and an observer (O) (Figure). Let the thickness of the lens vary as w(b) =w_0– b^2/α , where b is the vertical distance from the pole, w_0 is a constant. Using Fermat's principle, i.e., the time of transit fora ray between the source and observer is an extremum find the condition that all paraxial rays starting from the source will converge at a point O on the axis. Find the focal length.



(ii) A gravitational lens may be assumed to have a varying width of the form

$$w(b) = k_1 \ln\left(\frac{k_2}{b}\right) \qquad b_{min} < b < b_{max}$$

$$= k_1 \ln\left(\frac{k_2}{b_{min}}\right) \qquad b < b_{min}$$

Show that an observer will see an image of a point object as a ring about the centre of the lens with an angular radius.

$$\beta = \sqrt{\frac{(n-1)k_1 \frac{u}{v}}{u+v}}$$

Solution:

(i) The time taken by ray to travel from S to P_1 is

$$t_1 = \frac{SP_1}{c} = \frac{\sqrt{u^2 + b^2}}{c}$$

or $\quad t_1 = \frac{u}{c}\left(1 + \frac{1}{2}\frac{b^2}{u^2}\right)$ assuming $b << 1$.

The time required to travel from P_1 to O is

$$t_2 = \frac{P_1 O}{c} = \frac{\sqrt{v^2 + b^2}}{c} = \frac{v}{c}\left(1 + \frac{1}{2}\frac{b^2}{v^2}\right)$$

The time required to travel through the lens is

$$t_\ell = \frac{(n-1)w(b)}{c}$$

where n is the refractive index.

Thus, the total time is

$$t = \frac{1}{c}\left[u + v + \frac{1}{2}b^2\left(\frac{1}{u}+\frac{1}{v}\right) + (n-1)w(b)\right]$$

Put $\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$

Then, $t = \frac{1}{c}\left(u + v + \frac{1}{2}\frac{b^2}{D} + (n-1)\left(w_0 + \frac{b^2}{\alpha}\right)\right)$

Fermat's principle gives the time taken should be minimum.

For that first derivative should be zero.

$$\frac{dt}{db} = 0 = \frac{b}{CD} - \frac{2(n-1)b}{c\alpha}$$

$$\alpha = 2(n-1)D$$

Thus, a convergent lens is formed if $\alpha = 2(n-1)D$. This is independent of and hence all paraxial rays from S will converge at O i.e., for rays $b << n$ and $b << v$.

Since, $\frac{1}{D} = \frac{1}{u} + \frac{1}{v}$, the focal length is D.

(ii) In this case, differentiating expression of time taken t w.r.t. b.

$$t = \frac{1}{c}\left(u + v + \frac{1}{2}\frac{b^2}{D} + (n-1)k_1 \ln\left(\frac{k_2}{b}\right)\right)$$

$$\frac{dt}{db} = 0 = \frac{b}{D} - (n-1)\frac{k_1}{b}$$

$$\Rightarrow \quad b^2 = (n-1)k_1 D$$

$$\therefore \quad b = \sqrt{(n-1)k_1 D}$$

Thus, all rays passing at a height b shall contribute to the image. The ray paths make an angle.

$\beta = \frac{b}{v} = \frac{\sqrt{(n-1)k_1 D}}{v} = \sqrt{\frac{(n-1)k_1 uv}{v^2(u+v)}} = \sqrt{\frac{(n-1)k_1 u}{(u+v)v}}$. This is the required expression.