Chapter 8 - Electromagnetic Waves

Multiple Choice Questions (MCQs)
Single Correct Answer Type

Question 1. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in
(a) visible region (b) infrared region
(c) ultraviolet region (d) microwave region
Solution:

(c) Here it is given, the energy required to dissociate a carbon monoxide molecule into carbon and oxygen atoms is $E = 11$ eV

We know that, $E = hf$, where $h = 6.62 \times 10^{-34}$ J-s

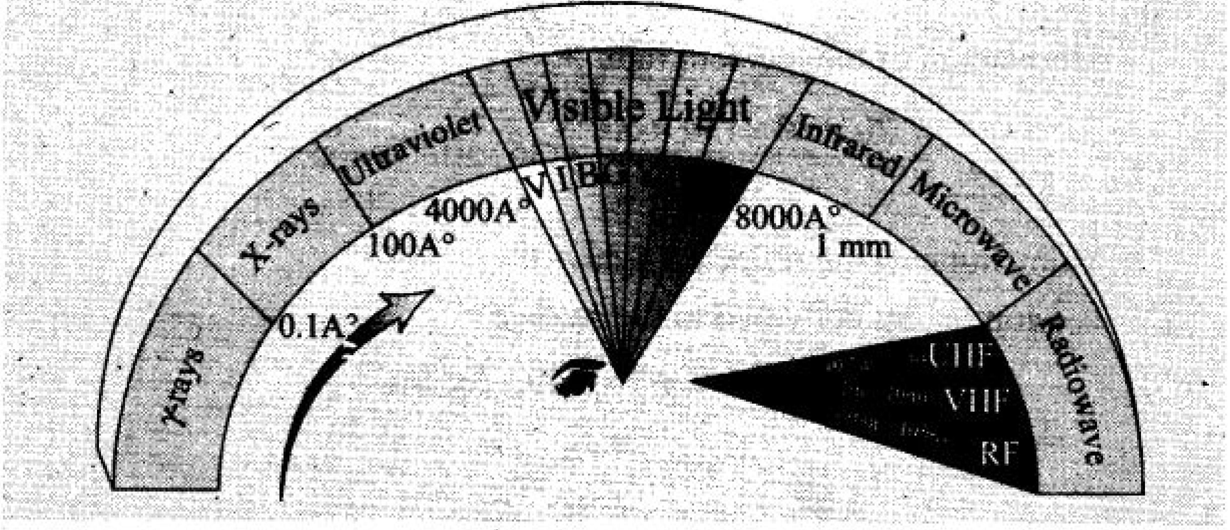
f = frequency

$$\Rightarrow \quad 11 \text{ eV} = hf$$

$$f = \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 2.65 \times 10^{15} \text{ Hz}$$

This frequency radiation belongs to ultraviolet region.

Important point:



Question 2.

A linearly polarised electromagnetic wave given as $E = E_0 \hat{i} \cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as

(a) $E_r = E_0 \hat{i} \ (kz - \omega t)$ (b) $E_r = E_0 \hat{i} \cos(kz + \omega t)$

(c) $E_r = -E_0 \hat{i} \cos(kz + \omega t)$ (d) $E_r = E_0 \hat{i} \sin(kz - \omega t)$

Solution: (b)

Key concept: When a wave is reflected from a denser medium or perfectly reflecting wall made with optically inactive material, then the type of wave doesn't change but only its phase changes by 180° or π radian.

Question 3. Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2, the total momentum delivered (for complete absorption) during 30 min is
(a) 36 x 10^{-5} kg-m/s (b) 36 x 10^{-4} kg-m/s
(c) 108 x 10^4 kg-m/s (d) 1.08 x 10^7 kg-m/s
Solution:

(b) Given, energy flux $\phi = 20 \text{ W/cm}^2$

Area, $A = 30 \text{ cm}^2$

Time, $t = 30 \text{ min} = 30 \times 60 \text{ s}$

Now, total energy falling on the surface in time t is,

$$U = \phi A t = 20 \times 30 \times (30 \times 60) \text{ J}$$

Momentum of the incident light $= \dfrac{U}{c}$

$$= \frac{20 \times 30 \times (30 \times 60)}{3 \times 10^8} = 36 \times 10^{-4} \text{ kg-ms}^{-1}$$

Momentum of the reflected light = 0

∴ Momentum delivered to the surface

$$= 36 \times 10^{-4} - 0 = 36 \times 10^{-4} \text{ kg-ms}^{-1}$$

Important points

Mass of photon:

Actually rest mass of the photon is zero. But its effective mass is given as $E = mc^2 = h\nu \Rightarrow m = \dfrac{E}{c^2} = \dfrac{h\nu}{c^2} = \dfrac{h}{c\lambda}$. This mass is also known as kinetic mass of the photon.

Momentum of the photon:

Momentum $p = m \times c = \dfrac{E}{c} = \dfrac{h\nu}{c} = \dfrac{h}{\lambda}$

Number of emitted photons:

The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as $(n) = \dfrac{P}{E} = \dfrac{P}{h\nu} = \dfrac{P\lambda}{hc}$; where E = energy of each photon

Intensity of light (I):

Energy crossing per unit area normally per second is called intensity or energy flux

i.e. $I = \dfrac{E}{At} = \dfrac{P}{A} \left(\dfrac{E}{t} = P = \text{radiation power} \right)$

At a distance r from a point source of power P intensity is given by $I = \dfrac{P}{4\pi r^2} \Rightarrow I \propto \dfrac{1}{r^2}$

Question 4. The electric field intensity produced by the radiations coming from a 100 W bulb at a 3 m distance is E. The electric field intensity produced by the radiations coming from 50

W bulb at the same distance is

(a) $\frac{E}{2}$ (b) $2E$

(c) $\frac{E}{\sqrt{2}}$ (d) $\sqrt{2}E$

Solution:

(c) We know the electric field intensity on a surface due to incident radiation is

$$I_{av} \propto E_0^2$$

$$\frac{P_{av}}{A} \propto E_0^2$$

Here $P_{av} \propto E_0^2$ [$\because$ A is same in both cases]

We know that, $E_0 \propto \sqrt{P_{av}}$

$$\therefore \quad \frac{(E_0)_1}{(E_0)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}} \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{E}{(E_0)_2} = \sqrt{\frac{1000}{5}}$$

$$(E_0)_2 = E/\sqrt{2}$$

Now according to question, $P' = 50$ W, $P = 100$ W

$\therefore$ Putting these value in Eq. (i), we get

$$\frac{E'}{E} = \frac{50}{100} \Rightarrow \frac{E'}{E} = \frac{1}{2} \Rightarrow E' = \frac{E}{2}$$

Question 5.

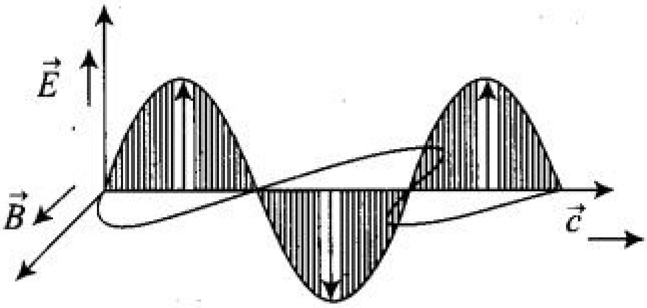
If $\vec{E}$ and $\vec{B}$ represent electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along

(a) $\vec{E}$ (b) $\vec{B}$

(c) $\vec{B} \times \vec{E}$ (d) $\vec{E} \times \vec{B}$

Solution: (d)

Key concept: A changing electric field produces a changing magnetic field and vice versa which gives rise to a transverse wave known as electromagnetic wave. The time varying electric and magnetic field are mutually perpendicular to each other and also perpendicular to the direction of propagation of this wave. The electric vector is responsible for the optical effects of an EM wave and is called the light vector.



The direction of propagation of electromagnetic wave is perpendicular to both electric field vector ($\vec{E}$) and $\vec{B}$ magnetic field vector B, i.e., in the direction of $\vec{E} \times \vec{B}$.

Here, electromagnetic wave is along the z-direction which is given by the cross product of E and B.

Question 6. The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is

(a) $c : 1$ (b) $c^2 : 1$

(c) $1 : 1$ (d) $\sqrt{c} : 1$

Solution:

(c) The intensity of electromagnetic wave is given by,

$I = U_{av}c$, where U_{av} = Average energy and c = speed of light

Intensity in relation with electric field $U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$

Intensity relation with magnetic field $U_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0}$

Now taking the intensity in terms of electric field,

$$(U_{av})_{\text{electric field}} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 (cB_0)^2 \qquad (\because E_0 = cB_0)$$

But, $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\therefore \qquad (U_{av})_{\text{Electric field}} = \frac{1}{2}\varepsilon_0 \times \frac{1}{\mu_0 \varepsilon_0} B_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

$$= (U_{av})_{\text{magnetic field}}$$

Hence the energy in electromagnetic wave is divided equally between electric field vector and magnetic field vector.

It means the ratio of contributions by the electric field and magnetic field components to the intensity of an electromagnetic wave is 1:1.

Important points: -

Properties of EM Waves

Speed: In free space, its speed $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E_0}{B_0} = 3 \times 10^8$ m/s.

In medium $v = \frac{1}{\sqrt{\mu \varepsilon}}$; where μ_0 = Absolute permeability, ε_0 = Absolute permittivity, E_0 and B_0 = Amplitudes of electric field and magnetic field vectors.

Energy: The energy in an EM waves is divided equally between the electric and magnetic fields.

Energy density of electric field $u_e = \frac{1}{2}\varepsilon_0 E^2$, Energy density of magnetic field $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

It is found that $u_e = u_B$. Also $u_{av} = u_e + u_B = 2u_e = 2u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$

Intensity (I): The energy crossing per unit area per unit time, perpendicular to the direction of propagation of EM wave is called intensity. $I = u_{av} \times c = \frac{1}{2}\varepsilon_0 E^2 c = \frac{1}{2}\frac{B^2}{\mu_0} \cdot c$

Momentum: EM waves also carries momentum, if a portion of EM wave of energy u propagating with speed c, then linear momentum $= \frac{\text{Energy } (u)}{\text{Speed } (c)}$

- When the incident EM wave is completely absorbed by a surface, it delivers energy u and momentum u/c to the surface.
- When a wave of energy u is totally reflected from the surface, the momentum delivered to surface is 2u/c.

Poynting vector $(\vec{S})$**:** In EM waves, the rate of flow of energy crossing a unit area is described by the poynting vector. Its unit is watt/m^2 and $\vec{S} = \frac{1}{\mu_o}(\vec{E} \times \vec{B}) = c^2\varepsilon_0(\vec{E} \times \vec{B})$. Because in EM waves, $\vec{E}$ and $\vec{B}$ are perpendicular to each other, the magnitude of $\vec{S}$ is $|\vec{S}| = \frac{1}{\mu_0} E B \sin 90° = \frac{EB}{\mu_0} = \frac{E^2}{\mu C}$.

- **The direction of the Poynting vector $\vec{S}$ at any point gives the wave's direction of travel and direction of energy transport the point.**

Radiation pressure: Is the momentum imparted per second per unit area on which the light falls.

For a perfectly reflecting surface $P_r = \frac{2S}{c}$; S = Poynting vector; c = Speed of light

For a perfectly absorbing surface $P_a = \frac{S}{c}$.

- **The radiation pressure is real that's why tails of comet point away from the sun.**

Question 7. An EM wave radiates outwards from a dipole antenna, with E_0 as the amplitude of its electric field vector. The electric field E_0 which transports significant energy from the source falls off as

(a) $\frac{1}{r^3}$

(b) $\frac{1}{r^2}$

(c) $\frac{1}{r}$

(d) remains constant

Solution: (c) A diode antenna radiates the electromagnetic waves outwards. The amplitude of electric field vector (E_0) which transports significant energy from the source falls intensity inversely as the distance (r) from the antenna,

i.e., $E_0 \propto \frac{1}{r}$.

One or More Than One Correct Answer Type

Question 8.

An electromagnetic wave travels in vacuum along z-direction $\vec{E} = (E_1\hat{i} + E_2\hat{j}) \cos(kz - \omega t)$. Choose the correct options from the following:

(a) The associated magnetic field is given as

$$\vec{B} = \frac{1}{c}(E_1\hat{i} - E_2\hat{j}) \cos(kz - \omega t)$$

(b) The associated magnetic field is given as

$$\vec{B} = \frac{1}{c}(E_1\hat{i} - E_2\hat{j}) \cos(kz - \omega t)$$

(c) The given electromagnetic field is circularly polarised.

(d) The given electromagnetic wave is plane polarised.

Solution: (a, d) We are given that the electric field vector of an electromagnetic wave travels in a vacuum along z-direction as,

$$\vec{E} = (E_2\hat{i} + E_2\hat{j}) \cos(kz - \omega t)$$

The magnitude of the electric and the magnetic fields in an electromagnetic wave are related as

$$B_0 = \frac{E_0}{c}$$

$$\vec{B} = \frac{\vec{E}}{c} = \frac{E_1\hat{i} + E_2\hat{j}}{c} \cos(kz - \omega t)$$

Also, $\vec{E}$ and $\vec{B}$ are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to $\vec{E}$ as well as $\vec{B}$, so the given electromagnetic wave is plane polarized.

Question 9.

An electromagnetic wave travelling along z-axis is given as $E = E_0 \cos(kz - \omega t)$. Choose the correct options from the following:

(a) The associated magnetic field is given as $\vec{B} = \frac{1}{c}\hat{k} \times \vec{E} = \frac{1}{\omega}(\hat{k} \times \vec{E})$

(b) The electromagnetic field can be written in terms of the associated magnetic field as $\vec{E} = c(\vec{B} \times \hat{k})$

(c) $\hat{k} \cdot \vec{E} = 0, \hat{k} \cdot \vec{B} = 0$

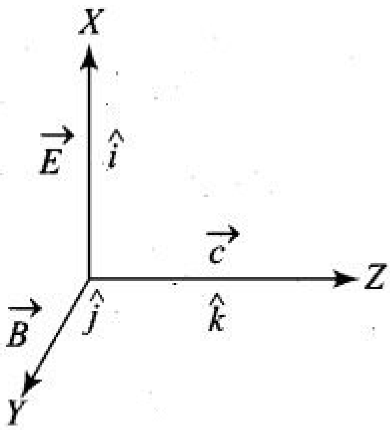
(d) $\hat{k} \times \vec{E} = 0, \hat{k} \times \vec{B} = 0$

Solution: (a, b, c)

(a) The direction of propagation of an electromagnetic wave is always along the direction of vector product $\vec{E} \times \vec{B}$. Refer to Figure.

$$\vec{B} = B\hat{j} = B(\hat{k} \times \hat{i}) = \frac{E}{c}(\hat{k} \times \hat{i})$$

$$= \frac{1}{c}[k \times E\hat{i}] = \frac{1}{c}[\hat{k} \times \vec{E}] \quad \left(\text{as } \frac{E}{B} = c\right)$$



(b) $\vec{E} = E\hat{i} = cB(\hat{j} \times \hat{k}) = c(B\hat{j} \times \hat{k}) = c(\vec{B} \times \hat{k})$

(c) $\hat{k} \cdot \vec{E} = \hat{k} \cdot (E\hat{i}) = 0, \hat{k} \cdot \vec{B} = \hat{k} \cdot (B\hat{j}) = 0$

(d) $\hat{k} \times \vec{E} = \hat{k} \times (E\hat{i}) = E(\hat{k} \times \hat{i}) = E\hat{j}$

and $\hat{k} \times \vec{B} = \hat{k} \times (B\hat{j}) = B(\hat{k} \times \hat{j}) = -B\hat{i}$

Question 10. A plane electromagnetic wave propagating along x-direction can have the following pairs of E and B.
(a) E_x, B_y (b) E_y, B_z
(c) B_x,E_y (d) E_z,B_y
Solution: (b, d)

Key concept: The direction of propagation of electromagnetic wave is perpendicular to both electric field vector ($\vec{E}$) and $\vec{B}$ magnetic field vector B, i.e., in the direction of $\vec{E} \times \vec{B}$.

Here in the question electromagnetic wave is propagating along x-direction. So, electro and magnetic field vectors should have either y-direction or 2-direction.

Question 11. A charged particle oscillates about its mean equilibrium position with a frequency of 10^9 Hz. The electromagnetic waves produced
(a) will have frequency of 10^9 Hz
(b) will have frequency of 2 x 10^9 Hz
(c) will have wavelength of 0.3 m
(d) fall in the region of radio waves
Solution: (a, c, d)
Here we are given the frequency by which the charged particles oscillates about its mean equilibrium position, it is equal to 10^9 Hz. The frequency of electromagnetic waves produced by a charged particle is equal to the frequency by which it oscillates about its mean equilibrium position.
So, frequency of electromagnetic waves produced by the charged particle is v= 10^9 Hz.

Wavelength $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{10^9} = 0.3$ m

The frequency of 10^9 Hz falls in the region of radiowaves.

Question 12. The source of electromagnetic waves can be a charge
(a) moving with a constant velocity
(b) moving in a circular orbit
(c) at rest
(d) falling in an electric field

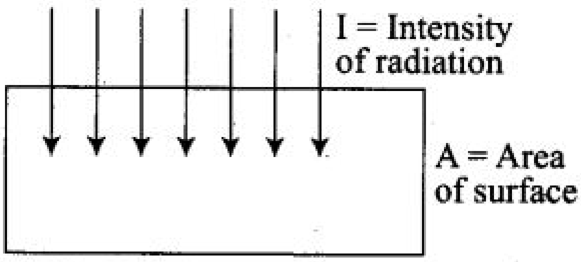
Solution: (b, d)
Key concept:

- An electromagnetic wave can be produced by accelerated or oscillating charge.
- An oscillating charge is accelerating continuously, it will radiate electromagnetic waves continuously.
- Electromagnetic waves are also produced when fast moving electrons are suddenly stopped by a metal target of high atomic number.

Here, in option (b) charge is moving in a circular orbit.
In circular motion, the direction of the motion of charge is changing continuously, thus it is an accelerated motion and this option is correct.
In option (d), the charge is falling in electric field. If a charged particle is moving in electric field it experiences a force or we can say it accelerates. We know an accelerating charge particle radiates electromagnetic waves. Hence option (d) is also correct.
Also, we know that a charge starts accelerating when it falls in an electric field.
Important points:

- In an atom an electron is circulating around the nucleus in a stable orbit, although accelerating does not emit electromagnetic waves; it does so only when it jumps from a higher energy orbit to a lower energy orbit.
- A simple LC oscillator and energy source can produce waves of desired frequency

Question 13. An EM wave of intensity I falls on a surface kept in vacuum and exerts radiation pressure p on it. Which of the following are true?
(a) Radiation pressure is I/c if the wave is totally absorbed
(b) Radiation pressure is —I/c if the wave is totally reflected
(c) Radiation pressure is 2I/c if the wave is totally reflected
(d) Radiation pressure is in the range I/c < p < 2I/c for real surfaces
Solution: (a, c, d)
Key concept: Radiation pressure (p) is the force exerted by electromagnetic wave on unit area of the surface, i.e., rate of change of momentum per unit area of the surface.
Let us consider a surface exposed to electromagnetic radiation as shown in figure. The radiation is falling normally on the surface. Further, intensity of radiation is I and area of surface exposed to radiation is A.



E = Energy received by surface per second = $I.A$

N = Number of photons received by surface per second

$$N = \frac{E}{E_{\text{photon}}} = \frac{E\lambda}{hc} = \frac{IA\lambda}{hc}$$

Now, there are three cases possible which are as follows:

Case I: Surface is perfectly reflecting

$$\Delta P_{\text{one photon}} = \text{Change in momentum} = \frac{2h}{\lambda}$$

$\therefore$ Total force experienced $F = N \times \Delta P_{\text{one photon}} = \dfrac{2IA}{c}$

Also, Pressure $P = \dfrac{F}{A} = \dfrac{2I}{c}$

Case II: Surface is perfectly absorbing

$$\Delta P_{\text{one photon}} = \frac{h}{\lambda}$$

$\Rightarrow$ $$F = N \times \Delta P_{\text{one photon}} = \frac{IA}{c}$$

Also, Pressure $P = \dfrac{F}{A} = \dfrac{I}{c}$

Hence radiation pressure is in the range $\dfrac{I}{c} < p < \dfrac{2I}{c}$ for real surfaces.

Important points:

If surface is partly reflecting

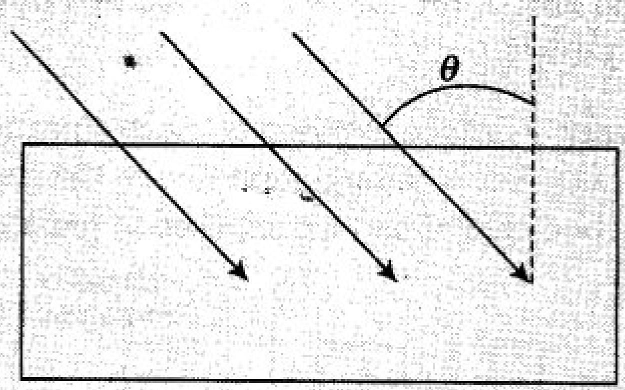
Let us consider that surface reflects 70 % and absorbs 30% of the incident radiation.

$$F = 0.7\left(\frac{2IA}{c}\right) + 0.3\left(\frac{IA}{c}\right) = \frac{1.7IA}{c}$$

Remarks:

(i) Radiation force/pressure supports photon theory of radiation.

(ii) If radiation falls obliquely, then appropriate projection of area vector is taken.



For situation as shown in figure,

$$F = \frac{2IA\cos^2\theta}{c}$$, for perfectly reflecting surface

$$F = \frac{IA\cos\theta}{c}$$, for perfectly absorbing surface

$$F = \frac{1.4\,IA\cos^2\theta}{c} + \frac{0.3\,IA\cos\theta}{c}$$, for partially reflecting surface

Very Short Answer Type Questions

Question 14. Why is the orientation of the portable radio with respect to broadcasting station important?

Solution: The electromagnetic waves are plane polarised, so the receiving antenna should be parallel to the vibration of the electric or magnetic field of the wave. So the receiving antenna should be parallel to electric/magnetic part of the wave. That is why the orientation of the portable radio with respect to broadcasting station is important.

Question 15. Why does microwave oven heats up a food item containing water molecules most efficiently?

Solution: The microwave oven heats up the food items containing water molecules most efficiently because the frequency of microwaves matches the resonant frequency of water molecules.

Question 16. The charge on a parallel plate capacitor varies as $q=q_0$ cos 2πvt. The plates are very large and close together (area = A, separation = d). Neglecting the edge effects, find the displacement current through the capacitor.
Solution:

The displacement current through the capacitor is given by,

$$I_d = I_c = \frac{dq}{dt} \quad \ldots(i)$$

Herę we are given, $q = q_0 \cos 2\pi vt$

Putting this value in Eq (i), we get

$$I_d = l_c = -q_0 \sin 2\pi vt \times 2\pi v$$

$$I_d = I_c = -2\pi v q_0 \sin 2\pi vt$$

Question 17. A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency?
Solution:

Capacitive reaction $X_C = \frac{1}{2\pi fC}$

Hence, $X_C \propto \frac{1}{f}$

As frequency decreases, X_C increases and the conduction current is inversely proportional to $X_C \left(\because I \propto \frac{1}{X_C}\right)$.

It means the displacement current decreases as the conduction current is equal to the displacement current.

Question 18. The magnetic field of a beam emerging from a fitter facing a flood light is given by B_0= 12 x 10^{-8} sin (1.20 x 10^7z- 3.60 x 10^{15} t) T What is the average intensity of the beam?
Solution:

The standard equation of magrietic field can be expressed as $B = B_0 \sin \omega t$.
We are given equation

$$B = 12 \times 10^{-8} \sin (120 \times 10^7 z - 3.60 \times 10^{1/5} t)\text{T}$$

On comparing this equation with standard equation, we get

$$B_0 = 12 \times 10^{-8}\text{ T}$$

The average intensity of the beam

$$I_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0} \cdot c = \frac{1}{2} \times \frac{(12 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}}$$

$$= 1.71 \text{ W/m}^2$$

Question 19.

Poynting vectors S is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction of wave propagation.

Mathematically, it is given by $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$. Show the nature of S versus t graph.

Solution:

In an electromagnetic waves, let $\vec{E}$ be varying along y-axis, $\vec{B}$ is along z-axis and propagation of wave be along x-axis. Then $\vec{E} \times \vec{B}$ will tell the direction of propagation of energy flow in electromagnetic wave, along x-axis.

Let $\quad \vec{E} = E_0 \sin(\omega t - kx)\hat{j}$

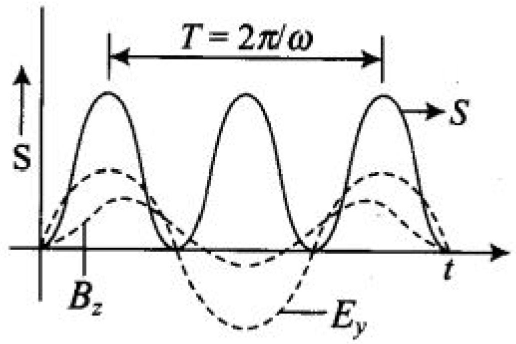
$\vec{B} = B_0 \sin(\omega t - kx)\hat{k}$

$$S = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{\mu_0} E_0 B_0 \sin^2(\omega t - kx)(\hat{j} \times \hat{k})$$

$$\Rightarrow \quad S = \frac{E_0 B_0}{\mu_0} \sin^2(\omega t - kx)\hat{i} \quad (\text{As } \hat{j} \times \hat{k} = \hat{i})$$

Since $\sin^2(\omega t - kx)$ is never negative, $\vec{S}(x, t)$ always points in the positive X-direction, i.e, in the direction of wave propagation.

The variation of $|S|$ with time T will be as given in the figure below:



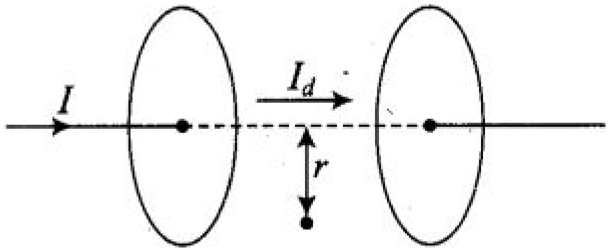
Question 20. Professor CV Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

Solution: The properties of an electromagnetic wave is same as other waves. Like other wave an electromagnetic wave also carries energy and momentum. Since, it carries momentum, an electromagnetic wave also exerts pressure called radiation pressure. This property of electromagnetic waves helped professor C V Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it.

Short Answer Type Questions

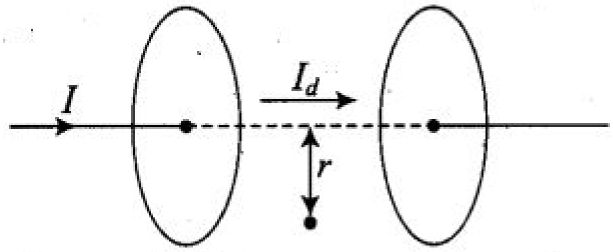
Question 21.

Show that the magnetic field B at a point in between the plates of a parallel plate capacitor during charging is $\frac{\mu_0 \varepsilon_r}{2} \frac{dE}{dt}$ (symbols having usual meaning).



Solution:

Let us assume I_d be the displacement current in the region between two plates of parallel plate capacitor, in the figure.



The magnetic field at a point between two plates of capacitor at a perpendicular distance r from the axis of plates is given by

$$B = \frac{\mu_0 2I_d}{4\pi r} = \frac{\mu_0}{2\pi r} I_d = \frac{\mu_0}{2\pi r} \times \varepsilon_r \frac{d\phi_E}{dt} \qquad \left[\because I_d = \frac{E_0 d\phi_E}{dt}\right]$$

$$\Rightarrow \qquad B = \frac{\mu_0 \varepsilon_r}{2\pi r} \frac{d}{dt}(E\pi r^2) = \frac{\mu_0 \varepsilon_r}{2\pi r} \pi r^2 \frac{dF}{dt}$$

$$\Rightarrow \qquad B = \frac{\mu_0 \varepsilon_r}{2} \frac{dE}{dt} \qquad \left[\because \phi_E = E\pi r^2\right]$$

Question 22. Electromagnetic waves with wavelength
(i) λ_1, is used in satellite communication.
(ii) λ_2, is used to kill germs in water purifier.
(iii) λ_3, is used to detect leakage of oil in underground pipelines.
(iv) λ_4, is used to improve visibility in runways during fog and mist conditions.
(a) Identify and name the part of electromagnetic spectrum to which these radiations belong.
(b) Arrange these wavelengths in ascending order of their magnitude.
(c) Write one more application of each.

Solution: (a) (i) In satellite communications, microwave is widely used. Hence λ_1, is the wavelength of microwave.
(ii) In water purifier, ultraviolet rays are used to kill germs. So, λ_2 is the wavelength of UV rays.
(iii) X-rays are used to detect leakage of oil in underground pipelines. So, λ_3 is the wavelength of X-rays.
(iv) Infrared rays are used to improve visibility on runways during fog and mist conditions. So, it is the wavelength of infrared waves.
(b) Wavelength of X-rays < wavelength of UV < wavelength of infrared < wavelength of microwave.

=> $\lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$

(c)

Radiation	Uses
γ-rays	Gives informations on nuclear structure, medical treatment etc.
X-rays	Medical diagnosis and treatment study of crystal structure, industrial radiograph.
UV-rays	Preserve food, sterilizing the surgical instruments, detecting the invisible writings, finger prints etc.
Visible light	To see objects
Infrared rays	To treat, muscular strain for taking photography during the fog, haze etc.
Micro wave and radio wave	In radar and telecommunication.

Question 23.

Show that average value of radiant flux density S over a single period T is given by $S = \frac{1}{2c\mu_0} E_0^2$.

Solution:

Radiant flux density

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = c^2\varepsilon_0(\vec{E} \times \vec{B}) \qquad \left[\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right]$$

Let electromagnetic waves be propagating along x-axis. If electric field vector of electromagnetic wave be along y-axis, then magnetic field vector be along z-axis. Therefore,

$$E = E_0 \cos(kx - \omega t)$$

and

$$B = B_0 \cos(kx - \omega t)$$

$$E \times B = (E_0 \times B_0)\cos^2(kx - \omega t)$$

$$S = c^2\varepsilon_0(E \times B)$$

$$= c^2\,\varepsilon_0(E_0 \times B_0)\cos^2(kx - \omega t)$$

Average value of the magnitude of radiant flux density over complete cycle is

$$S_{av} = c^2\varepsilon_0|E_0 \times B_0|\frac{1}{T}\int_0^T \cos^2(kx - \omega t)dt$$

$$\Rightarrow \quad S_{av} = c^2\varepsilon_0 E_0 B_0 \times \frac{1}{T} \times \frac{T}{2}$$

$$\text{As}\left[\int_0^T \cos^2(kx - \omega t)dt = \frac{T}{2}\right]$$

$$\Rightarrow \quad S_{av} = \frac{c^2}{2}\varepsilon_0 E_0\left(\frac{E_0}{c}\right) \qquad \left[\text{As } c = \frac{E_0}{B_0}\right]$$

$$\Rightarrow \quad S_{av} = \frac{c}{2}\varepsilon_0 E_0^2$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad \text{or} \quad \varepsilon_0 = \frac{1}{c^2\mu_0}$$

$$\Rightarrow \quad S_{av} = \frac{E_0^2}{2\mu_0 c}.$$ Hence proved.

Question 24. You are given a 2 μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates?

Solution:

The capacitance of capacitor $C = 2\mu F$,

Displacement current $I_d = 1$ mA

Charge in capacitor, $q = CV$

$$I_d dt = CdV \qquad [\because q = it]$$

or $$I_d = C\frac{dV}{dt}$$

$$1 \times 10^{-3} = 2 \times 10^{-6} \times \frac{dV}{dt}$$

or $$\frac{dV}{dt} = \frac{1}{2} \times 10^3 = 500 \text{ V/s}$$

Hence by applying a varying potential difference of 500 V/s, we would produce a displacement current of desired value.

Question 25. Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is I/C.

Solution: Let us consider a surface exposed to electromagnetic radiation. The radiation is falling normally on the surface. Further, intensity of radiation is I and area of surface exposed to radiation is A.

E = Energy received by surface per second = $I.A$

N = Number of photons received by surface per second

$$N = \frac{E}{E_{\text{photon}}} = \frac{E\lambda}{hc} = \frac{IA\lambda}{hc}$$

Let the surface is perfectly absorbing, $\Delta P_{\text{one photon}} = \frac{h}{\lambda}$.

$$\Rightarrow \qquad F = N \times \Delta P_{\text{one photon}} = \frac{IA}{c}$$

Also, $$\text{Pressure } P = \frac{F}{A} = \frac{I}{c}$$

Question 26. What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of room, its intensity essentially remains constant.
What geometrical characteristic of LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?

Solution: We know intensity of light from a point source I α $1/r^2$, r is the distance from point source.

As the distance is doubled, so the intensity becomes one-fourth the initial value. But in case of laser it does not spread, so its intensity remain same.

Some geometrical characteristics of LASER beam which are responsible for the constant intensity is

(i) Unidirection (ii) Monochromatic

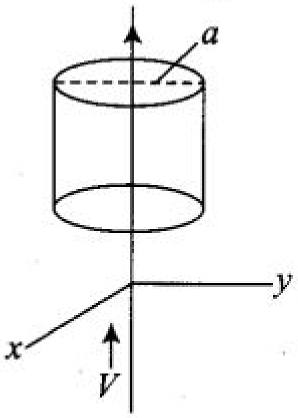
(iii) Coherent light (iv) Highly collimated

These characteristics are missing in the case of normal light from the bulb.

Question 27. Even though an electric field E exerts a force qE on a charged particle yet electric field of an EM wave does not contribute to the radiation pressure (but transfers energy). Explain.
Solution: The electric field of an electromagnetic wave is an oscillation field. It exerts electric force on a charged particle, but this electric force averaged over an integral number of cycles is zero, since its direction changes every half cycle. Hence, electric field is not responsible for radiation pressure though it transfer the energy. In fact, radiation pressure appears as a result of the action of the magnetic field of the wave on the electric currents induced by the electric field of the same wave.

Long Answer Type Questions
Question 28.

An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the z-axis (figure). The wire is set into motion along its length with a uniform velocity $\vec{v} = v\hat{\mathbf{k}}_z$. Calculate the pointing vector $\vec{S} = \frac{I}{\mu_0}(\vec{E} \times \vec{B})$.



Solution:

The electric field due to infinitely long thin wire

$$\vec{E} = \frac{\lambda \hat{e}_s}{2\pi\varepsilon_0 a}\hat{j}$$

Magnetic field due to the wire, $\vec{B} = \frac{\mu_0 i}{2\pi a}\hat{i}$

Equivalent current flowing through the wire, $i = \lambda v$

Hence $\vec{B} = \frac{\mu_0 \lambda v}{2\pi a}\hat{i}$

$$\therefore \qquad \vec{S} = \frac{1}{\mu_0}[\vec{E} \times \vec{B}] = \frac{1}{\mu_0}\left[\frac{\lambda}{2\pi\varepsilon_0 a}\hat{j} \times \frac{\mu_0 \lambda v}{2\pi a}\hat{i}\right]$$

$$\Rightarrow \qquad \vec{S} = \frac{\lambda^2 v}{4\pi^2 \varepsilon_0 a^2}(\hat{j} \times \hat{i}) = -\frac{\lambda^2 v}{4\pi^2 \varepsilon_0 a^2}\hat{k}$$

Question 29.

Sea water at frequency $v = 4 \times 10^8$ Hz has permeability $\varepsilon \approx 80\varepsilon_0$, permeability $\mu \approx \mu_0$ and resistivity $\rho = 0.25$ m. Imagine a parallel plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_0 \sin(2\pi vt)$. What fraction of the conduction current density is the displacement current density?

Solution:

Let the separation between the plates of capacitor immersed in sea water plates is d and applied voltage across the plates is $V(t) = V_0 \sin(2\pi vt)$.

Thus, electric field, $E = \dfrac{V(t)}{d}$

$$\Rightarrow \quad E = \frac{V_0}{d} \sin(2\pi vt)$$

Now using Ohm's law, the conduction current density $J^c = \dfrac{E}{\rho} = \dfrac{1}{\rho}\dfrac{V_0}{d} \sin(2\pi vt)$

$$\Rightarrow \quad J^c = \frac{V_0}{\rho d} \sin(2\pi vt) = J_0^c \sin 2\pi vt$$

Here, $\quad J_0^c = \dfrac{V_0}{\rho d}$

The displacement current density is given as

$$J^d = \varepsilon \frac{dE}{dt} = \varepsilon \frac{d}{dt}\left[\frac{V_0}{d} \sin(2\pi vt)\right]$$

$$= \frac{\varepsilon 2\pi v V_0}{d} \cos(2\pi vt)$$

$$\Rightarrow \quad J^d = J_0^d \cos(2\pi vt)$$

where, $\quad J_0^d = \dfrac{2\pi v \varepsilon V_0}{d}$

$$\Rightarrow \quad \frac{J_0^d}{J_0^c} = \frac{\dfrac{2\pi v \varepsilon V_0}{d}}{\dfrac{V_0}{\rho d}} = 2\pi v \varepsilon \rho$$

$$= 2\pi \times 80\varepsilon_0 v \times 0.25 = 4\pi\varepsilon_0 v \times 10$$

$$\Rightarrow \quad \frac{J_0^d}{J_0^c} = \frac{10 \times 4 \times 10^8}{9 \times 10^9} = \frac{4}{9}$$

Question 30.

A long straight cable of length l is placed symmetrically along z-axis and has radius $a(<< l)$. The cable consists of a thin wire and a co-axial conducting tube. An alternating current $I(t) = I_0 \sin(2\pi\nu t)$ flows down the central thin wire and returns along the co-axial conducting tube. The induced electric field at a distance s from the wire inside the cable is $\vec{E}(s,t) = \mu_0 I_0 \nu \cos(2\pi\nu t) \ln\left(\frac{s}{a}\right)\hat{k}$.

(i) Calculate the displacement current density inside the cable.
(ii) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I^d.
(iii) Compare the conduction current I_0 with the displacement current I_0^d.
Solution:

(i) We are given, the induced electric field at a distance r from the wire inside the cable.

$$\vec{E}(s,t) = \mu_0 I_0 \nu \cos(2\pi\nu t) \ln\left(\frac{s}{a}\right)\hat{k}$$

The displacement current density is given by $\vec{J}_d = \varepsilon_0 \frac{d\vec{E}}{dt}$

$$= \varepsilon_0 \frac{d}{dt}\left[\mu_0 I_0 \nu \cos(2\pi\nu t) \ln\left(\frac{s}{a}\right)\hat{k}\right]$$

$$= \varepsilon_0 \mu I_0 \nu \frac{d}{dt}[\cos 2\nu t] \ln\left(\frac{s}{a}\right)\hat{k}$$

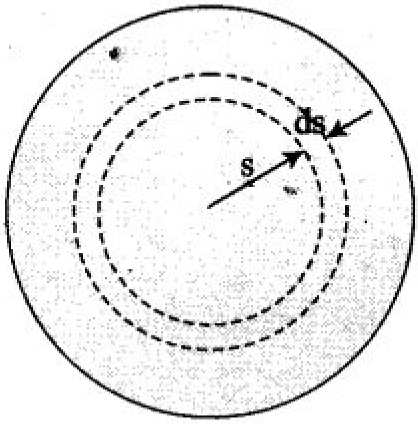
$$= \frac{1}{c^2} I_0 \nu^2 2\pi[-\sin 2\pi\nu t] \ln\left(\frac{s}{a}\right)\hat{k}$$

$$= \frac{\nu^2}{c^2} 2\pi I_0 \sin 2\pi\nu t \ln\left(\frac{a}{s}\right)\hat{k}$$

$$= \frac{1}{\lambda^2} 2\pi I_0 \ln\left(\frac{a}{s}\right) \sin 2\pi\nu t\,\hat{k}$$

$$\Rightarrow \quad \vec{J}_d = \frac{2\pi I_0}{\lambda^2} \ln\frac{a}{s} \sin 2\pi\nu t\,\hat{k}$$

(ii) Total displacement current, $I^d = \int J_d \, 2\pi s ds$



$$I^d = \int_0^a \left(\frac{2\pi I_0}{\lambda^2} \ln \frac{a}{s} \sin 2\pi v t \right) 2\pi s ds$$

$$= \int_0^a \left[\frac{2\pi}{\lambda^2} I_0 \int_{s=0}^a \ln\left(\frac{a}{s}\right) s ds \sin 2\pi v t \right] \times 2\pi$$

$$= \left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_0^a \ln\left(\frac{a}{s}\right) \frac{1}{2} d(s^2) \cdot \sin 2\pi v t$$

$$= \left(\frac{a}{2}\right)^2 \left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi v t \int_0^a \ln\left(\frac{a}{s}\right) \cdot d\left(\frac{s}{a}\right)^2$$

$$= \frac{a^2}{4} \left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi v t \int_0^a \ln\left(\frac{a}{s}\right)^2 \cdot d\left(\frac{s}{a}\right)^2$$

$$= \frac{a^2}{4} \left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi v t \times (1) \qquad \left[\because \int_{s=0}^a \ln\left(\frac{s}{a}\right)^2 d\left(\frac{s}{a}\right)^2 = 1 \right]$$

$$\therefore \quad I^d = \frac{a^2}{4} \left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi v t$$

$$\Rightarrow \quad I^d = \left(\frac{\pi a}{\lambda}\right)^2 I_0 \sin 2\pi v t$$

(iii) The displacement current,

$$I_d = \left(\frac{\pi a}{\lambda}\right)^2 I_0 \sin 2\pi v t = I_0^d \sin 2\pi v t$$

Here, $I_0^d = \left(\frac{a\pi}{\lambda}\right)^2 I_0$

$$\Rightarrow \quad \frac{I_0^d}{I_0} = \left(\frac{a\pi}{\lambda}\right)^2$$

Question 31.

A plane EM wave travelling in vacuum along z-direction is given by $E = E_0 \sin(kz - \omega t)\hat{i}$ and $B = B_0 \sin(kz - \omega t)\hat{j}$

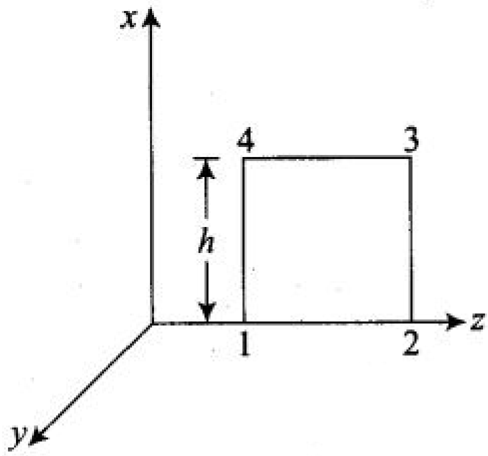
(i) Evaluate $\int E \cdot dl$ over the rectangular loop 1234 shown in figure.

(ii) Evaluate $\int B \cdot ds$ over the surface bounded by loop 1234.

(iii) Use equation $\oint E \cdot dl = \frac{-d\phi_B}{dt}$ to prove $\frac{E_0}{B_0} = c$.

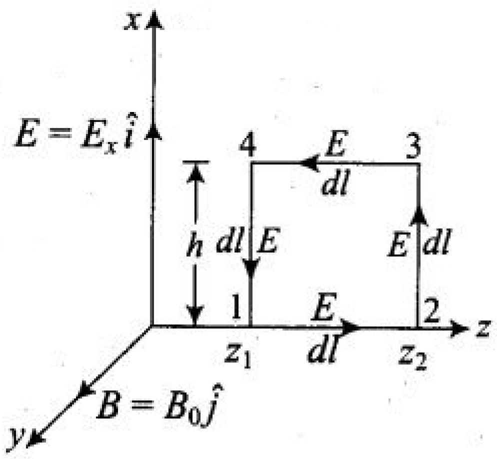
(iv) By using similar process and the equation

$\oint B \cdot dl = \mu_0 I + \varepsilon_0 \frac{d\phi_E}{dt}$, prove that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.



Solution:

(i) Let electromagnetic wave is propagating along z-axis, in this case electric field vector ($\vec{E}$) be along x-axis and magnetic field vector ($\vec{B}$) along y-axis, *i.e.*, $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{j}$.



The line integral of $\vec{E}$ over the closed rectangular path 1234 in x-z plane of the figure is

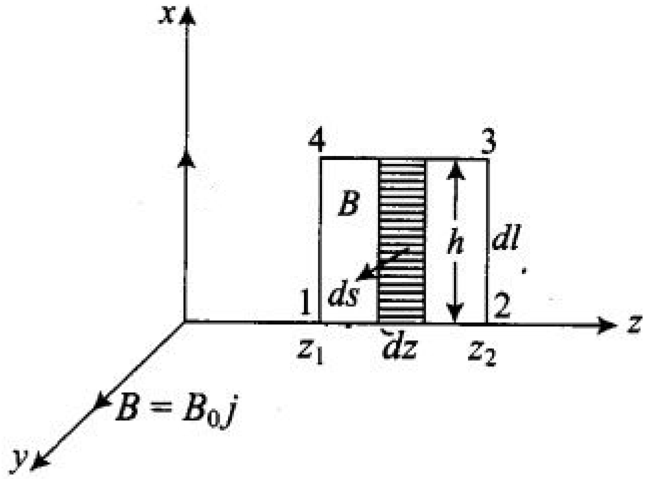
$$\oint \vec{E} \cdot d\vec{l} = \int_1^2 \vec{E} \cdot d\vec{l} + \int_2^3 \vec{E} \cdot d\vec{l} + \int_3^4 \vec{E} \cdot d\vec{l} + \int_4^1 \vec{E} \cdot d\vec{l}$$

$$= \int_1^2 E \cdot dl \cos 90° + \int_2^3 E \cdot dl \cos 0°$$

$$+ \int_3^4 E \cdot dl \cos 90° + \int_4^1 E \cdot dl \cos 180°$$

$$\oint \vec{E} \cdot d\vec{l} = E_0 h[\sin (kz_2 - \omega t) - \sin (kz_1 - \omega t)] \qquad \text{...(i)}$$

(ii) Now let us evaluate $\int \vec{B} \cdot d\vec{s}$, let us consider the rectangle 1234 to be made of strips of are $ds = hdz$ each.



$$\int \vec{B} \cdot d\vec{s} = \int B \cdot ds \cos 0 = \int B \cdot ds$$

$$= \int_{Z_1}^{Z_2} B_0 \sin (kz - \omega t) h dz$$

$$\int \vec{B} \cdot d\vec{s} = \frac{-B_0 h}{k} [\cos (kz_2 - \omega t) - \cos (kz_1 - \omega t)] \qquad \ldots\text{(ii)}$$

(iii) We are given $\oint E \cdot dl = \frac{-d\phi_B}{dt} = -\frac{d}{dt} \oint B \cdot ds$

Substituting the values from Eqs. (i) and (ii), we get

$$E_0h[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)]$$

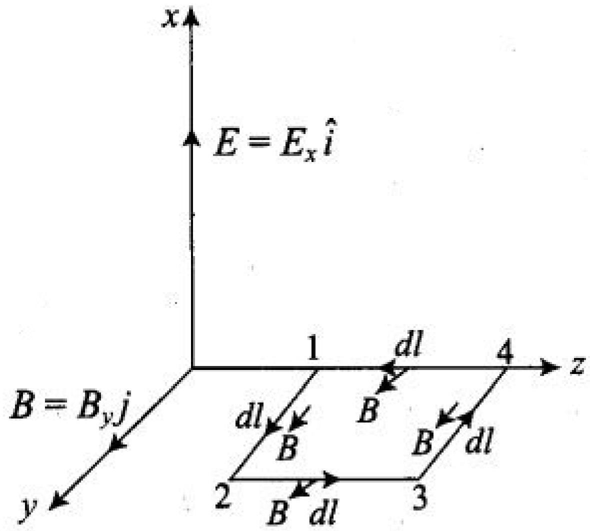
$$=\frac{-d}{dt}\left[\frac{B_0h}{k}\{\cos(kz_2-\omega t)-\cos(kz_1-\omega t)\right]$$

$$=\frac{B_0h}{k}\omega[\sin(kz_2-\omega t)-\sin(kz_1-\omega t)]$$

$$\Rightarrow \qquad E_0=\frac{B_0\omega}{k}=B_0c \qquad \left(\because \frac{\omega}{k}=c\right)$$

$$\Rightarrow \qquad \frac{E_0}{B_0}=c$$

(iv) For evaluating $\oint \vec{B}\cdot d\vec{l}$, let us consider a loop 1234 in y-z plane as shown in figure given below.



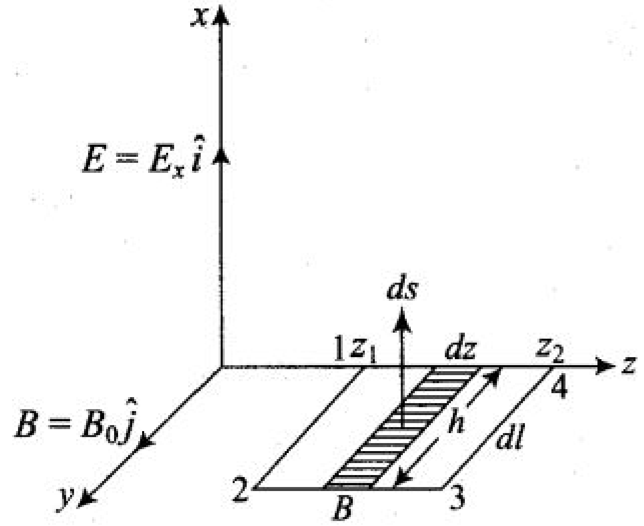
$$\oint \vec{B} \cdot d\vec{l} = \int_1^2 \vec{B} \cdot d\vec{l} + \int_2^3 \vec{B} \cdot d\vec{l} + \int_3^4 \vec{B} \cdot \vec{dl} + \int_4^1 \vec{B} \cdot d\vec{l}$$

$$= \int_1^2 B \cdot dl \cos 0° + \int_2^3 B \cdot dl \cos 90°$$

$$+ \int_3^4 B \cdot dl \cos 180° + \int_4^1 B \cdot dl \cos 90°$$

$$\oint \vec{B} \cdot d\vec{l} = B_0 h[\sin (kz_2 - \omega t) - \sin (kz_1 - \omega t)] \qquad \text{...(iii)}$$

Now to evaluate $\phi_E = \int \vec{B} \cdot d\vec{s}$, let us consider the rectangle 1234 to be made of strips of area *hds* each.



$$\phi_E = \int \vec{E} \cdot d\vec{s} = \int E ds \cos 0 = \int E ds$$

$$= \int_{z_1}^{z} E_0 \sin(kz_1 - \omega t) h dz$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{E_0 h}{k} [\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)]$$

$$\therefore \quad \frac{d\phi_E}{dt} = \frac{E_0 h \omega}{k} [\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad \text{...(iv)}$$

Let $\oint B \cdot dl = \mu_0 \left(I + \frac{\varepsilon_o d\phi_E}{dt} \right)$ where I = conduction current

= 0 in vacuum

$$\therefore \oint B \cdot dl = \mu_0 \varepsilon \frac{d\phi_E}{dt}$$

Using relations obtained in Eqs. (iii) and (iv) and simplifying, we get

$$B_0 = E_0 \frac{\omega \mu_0 \varepsilon_0}{k}$$

$$\Rightarrow \quad \frac{E_0}{B_0} \frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0}$$

But $\frac{E_0}{B_0} = c$ and $\omega = ck \Rightarrow c \cdot c = \frac{1}{\mu_0 \varepsilon_0}$,

therefore $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Question 32.

A plane EM wave travelling along z-direction is described by $E = E_0 \sin(kz - \omega t)\hat{i}$ and $B = B_0 \sin(kz - \omega t)\hat{j}$. Show that

(i) the average energy density of the wave is given by

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

(ii) the time averaged intensity of the wave is given by

$$I_{av} = \frac{1}{2} c \varepsilon_0 E_0^2.$$

Solution: (i) Total energy carried by electromagnetic wave is due to electric field vector and magnetic field vector. In electromagnetic wave, E and B vary from point to point and from moment to moment.

The energy density due to electric field E is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

The energy density due to magnetic field B is

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Total energy density of electromagnetic wave

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

Let the EM wave be propagating along z-direction. The electric field vector and magnetic field vector be represented by

$$E = E_0 \sin(kz - \omega t)$$

$$B = B_0 \sin(kz - \omega t)$$

The values of E^2 and B^2 vary from point to point and from moment to moment. Hence, the effective values of E^2 and B^2 are their time averages over complete cycle.

We know, $$\left\langle \sin^2\theta \right\rangle = \frac{\int_0^{2\pi} \sin^2\theta\, d\theta}{2\pi} = \frac{1}{2}$$

and $$\left\langle \cos^2\theta \right\rangle = \frac{\int_0^{2\pi} \cos^2\theta\, d\theta}{2\pi} = \frac{1}{2}$$

Hence, the time average value of E^2 over complete cycle,

$$\left\langle E^2 \right\rangle = \frac{\int_0^{T} \left[E_0 \sin(kz - \omega t)\right]^2 dt}{T} = \frac{E_0^2}{2}$$

And, the time average value of B^2 over complete cycle,

$$\langle B^2 \rangle = \frac{\int_0^T [B_0 \sin(kz - \omega t)]^2 \, dt}{T} = \frac{B_0^2}{2}$$

The time average of energy density over complete cycle

$$u_{\text{av}} = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{2} + \frac{1}{2}\mu_0\left(\frac{B_0^2}{2}\right)$$

$$\Rightarrow \quad u_{\text{av}} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

(ii) We know $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E_0}{B_0}$

where μ_0 = Absolute permeability, ε_0 = Absolute permittivity, E_0 and B_0 = Amplitudes of electric field and magnetic field vectors

The time average of energy density due to magnetic field B is

$$u_B = \frac{1}{2}\frac{B_0{}^2}{\mu_0} = \frac{1}{4}\frac{(E_0^2/c^2)}{\mu_0}$$

$$= \frac{E_0^2}{4\mu_0} \times \mu_0 \varepsilon_0 = \frac{1}{4}\varepsilon_0 E_0^2$$

Hence, $u_B = u_E$; the time average of energy density due to magnetic field is equal to the time average of energy density due to electric field.

$$\Rightarrow \quad u_{\text{av}} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

Time average intensity of the wave

$$I_{\text{av}} = u_{\text{av}} c = \left(\frac{1}{2}\varepsilon_0 E_0^2\right) c = \frac{1}{2} c \varepsilon_0 E_0^2$$