Chapter 7 - Alternating Current

Multiple Choice Questions (MCQs)
Single Correct Answer Type

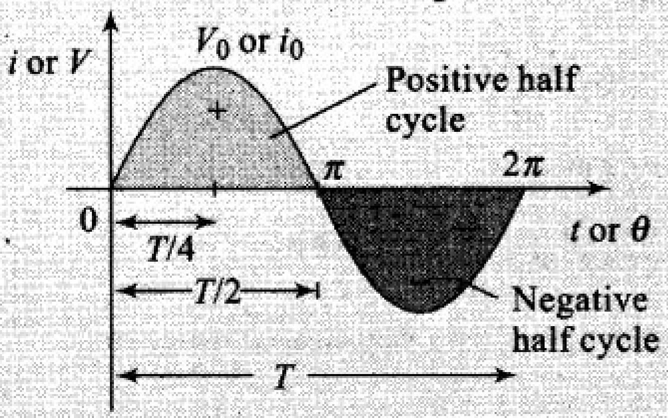
Question 1. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is

(a) $5\sqrt{2}$ A (b) $5\sqrt{\frac{3}{2}}$ A

(c) $\frac{5}{6}$ A (d) $\frac{5}{\sqrt{2}}$ A

Solution: (b)
Key concept: Equation for i and V: Alternating current or voltage varying as sine function can be written as

$$i = i_0 \sin \omega t = i_0 \sin 2\pi\nu t = i_0 \sin \frac{2\pi}{T} t$$



and $$V = V_0 \sin \omega t = V_0 \sin 2\pi\nu t = V_0 \sin \frac{2\pi}{T} t$$

where i and V are instantaneous values of current and voltage,
i_0 and V_0 are peak values of current and voltage
ω = Angular frequency in rad/sec, ν = Frequency in Hz and
T = time period

According to the problem, $f = 50$ Hz, $I_{rms} = 5$A

$$t = \frac{1}{300} \text{ s}$$

$$I_0 = \text{Peak value} = \sqrt{2}\,(I_{rms}) = 5\sqrt{2}$$

$$= 5\sqrt{2} \text{ A}$$

From, $$I = I_0 \sin \omega t = 5\sqrt{2} \sin 2\pi\nu t = 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{\frac{3}{2}} \text{ A}$$

Question 2. An alternating current generator has an internal resistance R_g and an internal reactance X_g It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to
(a) zero (b) X_g
(c) $-X_g$ (d) R_g
Solution: (c) For maximum power to be delivered from the generator (or internal reactance X_g) to the load (of reactance, X_L),
=> $X_L + X_g = 0$ (the total reactance must vanish)
=>$X_L=-X_g$

Question 3.
When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means
(a) input voltage cannot be AC voltage, but a DC voltage
(b) maximum input voltage is 220 V
(c) the meter reads not v but $<v^2>$ and is calibrated to read $\sqrt{<v^2>}$
(d) the pointer of the meter is stuck by some mechanical defect
Solution:
(c) The voltmeter connected to AC mains calibrated to read rms value $\sqrt{<v^2>}$.

Question 4. To reduce the resonant frequency in an L-C-R series circuit with a generator,
(a) the generator frequency should be reduced
(b) another capacitor should be added in parallel to the first
(c) the iron core of the inductor should be removed
(d) dielectric in the capacitor should be removed
Solution: (b)

Key concept: Resonant frequency (Natural frequency)

At resonance $X_L = X_C \Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C}$

$$\Rightarrow \qquad \omega_0 = \frac{1}{\sqrt{LC}} \frac{\text{rad}}{\text{sec}}$$

$$\Rightarrow \qquad \nu_0 = \frac{1}{2\pi\sqrt{LC}} \text{Hz}$$

Resonant frequency in an *L-C-R* circuit is given by

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

If L or C increases, the resonant frequency will reduce.

To increase capacitance, we must connect another capacitor parallel to the first.

Question 5. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication?
(a) R = 20 Ω, L = 1.5 H, C = 35μF
(b) R = 25 Ω, L = 2.5 H, C = 45 μF
(c) R=15Ω, L = 3.5H, C = 30 μF
(d) R = 25 Ω, L = 1.5 H, C = 45 μF
Solution: (c)

Key concept: Quality factor (Q-factor) of series resonant circuit:

(i) The characteristic of a series resonant circuit is determined by the quality factor (Q-factor) of the circuit.

(ii) It defines sharpness of i–v curve at resonance when Q-factor is large, the sharpness of resonance curve is more and vice-versa.

(iii) Q-factor is also defined as follows:

$$Q\text{-factor} = 2\pi \times \frac{\text{Max. energy stored}}{\text{Energy dissipation}}$$

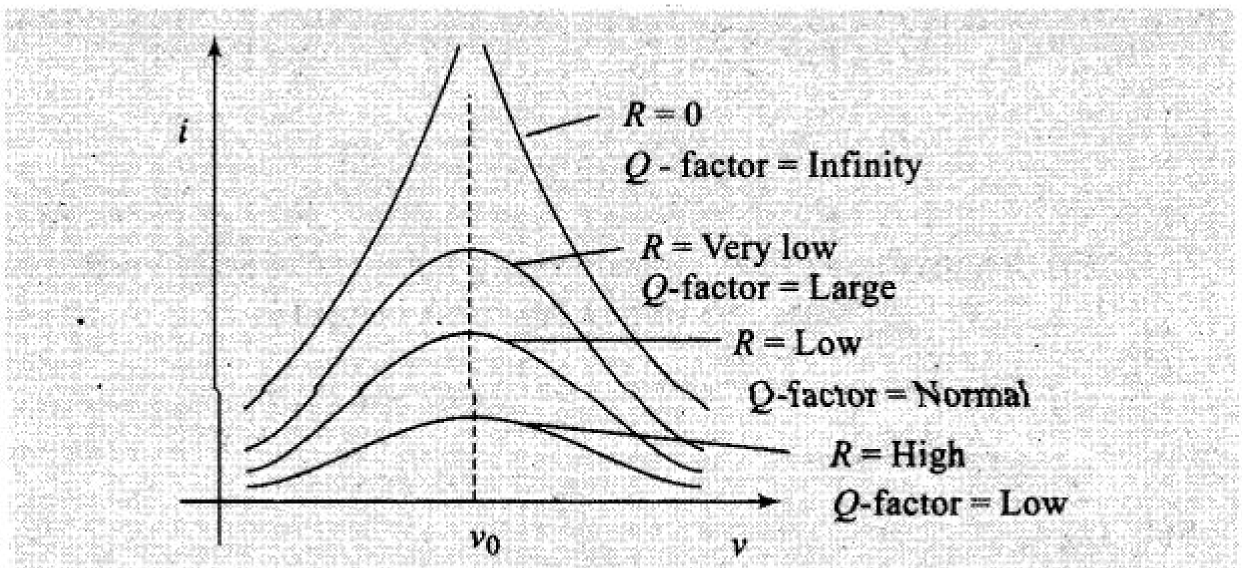
$$= \frac{2\pi}{T} \times \frac{\text{Max. energy stored}}{\text{Mean power dissipated}}$$

$$= \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega}$$

(iv) $$Q\text{-factor} = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R} = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR}$$

$$\Rightarrow \quad Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

For better tuning of an L-C-R circuit used for communication, quality factor of the circuit must be as high as possible.



Resonance curve

We know quality factor should be high for better tuning.

Quality factor (Q) of an L-C-R circuit is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where R is the resistance, L is the inductance and C is the capacitance of the circuit.
For high Q factor R should be low, L should be high and C should be low. These conditions are best satisfied by the values given in option (c).
Important point: Be careful while writing formula for quality factor, this formula we used in this case is only for series L-C-R circuit.

Question 6. An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6 V (rms) AC source. The power dissipated in the circuit is
(a) 8 W (b) 12 W
(c) 14.4 W (d) 18 W
Solution: (c) According to the problem, XL = 1 Ω , R = 2 Ω ,

$$E_{rms} = 6\text{ V}, P_{av} = ?$$

Average power dissipated in the circuit

$$P_{av} = E_{rms} I_{rms} \cos\phi \qquad \text{...(i)}$$

$$I_{rms} = \frac{E_{rms}}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$I_{rms} = \frac{6}{\sqrt{5}}\text{ A}$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \qquad \text{[from Eq. (i)]}$$

$$= \frac{72}{\sqrt{5}\sqrt{5}} = \frac{72}{5} = 14.4\text{ W}$$

Question 7. The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is

(a) $\frac{1}{\sqrt{2}}$ A

(b) $\sqrt{2}$ A

(c) 2 A

(d) $2\sqrt{2}$ A

Solution: (a)

Key concept: It decreases voltage and increases current

$V_S < V_P$

$N_S < N_P$

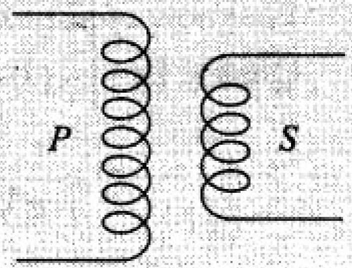
$E_S < E_P$

$i_S > i_P$

$R_S < R_P$

$t_S > t_P$

$k < 1$



According to the problem output/secondary voltage $V_S = 24$ V

Power associated with secondary $P_S = 12$ W

$$I_S = \frac{P_S}{V_S} = \frac{12}{24} = 0.5 \text{ A}$$

Amplitude of the current in the secondary winding

$$I_0 = I_S\sqrt{2}$$

$$= (0.5)(1.414) = 0.707 = \frac{1}{\sqrt{2}} \text{ A}$$

One or More Than One Correct Answer Type

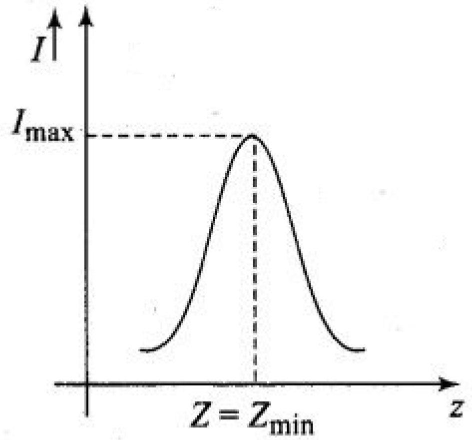
Question 8. As the frequency of an AC circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?

(a) Inductor and capacitor (b) Resistor and inductor

(c) Resistor and capacitor (d) Resistor, inductor and capacitor

Solution: (a, d) Compare the given circuit by predicting the variation in their reactances with frequency. So, that then we can decide the elements.

Reactance of an inductor of inductance L is $X_L = 2\pi vL$, where v is the frequency of the AC circuit.



X_c = Reactance of the capacitive circuit

$$= \frac{1}{2\pi fC}$$

With an increase in frequency (f) of an AC circuit, R remains constant, inductive reactance (X_L) increases and capacitive reactance (X_c) decreases.

For an L-C-R circuit,

Z = Impedance of the circuit

$$= \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + \left(2\pi vL - \frac{1}{2\pi vC}\right)^2}$$

As frequency (v) increases, Z decreases and at certain value of the frequency known as resonant frequency (v_0), impedance Z is minimum that is $Z_{min} = R$ current varies inversely with impedance and at Z_{min} current is maximum.

Question 9. In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit?
(a) Only resistor (b) Resistor and an inductor
(c) Resistor and a capacitor (d) Only a capacitor
Solution: (c, d) This is the similar problem as we discussed above. In this problem, the current increases on increasing the frequency of supply. Hence, the reactance of the circuit must be decreased as increase in frequency. So, one element that must be connected is capacitor. We can also connect a resistor in series.
For a capacitive circuit,
$X_C = 1/\omega C = 1/2\pi fC$

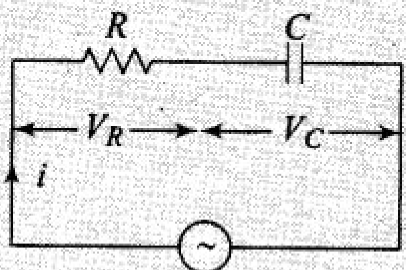
When frequency increases, X_C decreases. Hence current in the circuit increases.

Important point: Resistive, Capacitive Circuit (RC-Circuit)

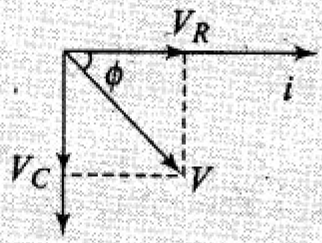
$V_R = iR,$

$V_C = iX_C,$

$V_R = iR$



$V = V_0 \sin \omega t$



(1) *Applied voltage:* $V = \sqrt{V_R^2 + V_C^2}$

(2) *Impedance:* $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

(3) *Current:* $i = i_0 \sin(\omega t + \phi)$

(4) *Peak current:* $i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_C^2}} = \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2 \nu^2 C^2}}}$

(5) *Phase difference:* $\phi = \tan^{-1} \frac{X_C}{R} = \tan^{-1} \frac{1}{\omega CR}$

(6) *Power factor:* $\cos \phi = \frac{R}{\sqrt{R^2 + X_C^2}}$

(7) *Leading quantity:* Current

Question 10. Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?
(a) For a given power level, there is a lower current
(b) Lower current implies less power loss
(c) Transmission lines can be made thinner
(d) It is easy to reduce the voltage at the receiving end using step-down transformers
Solution: (a, b, d)

Key concept: Power loss due to transmission lines having resistance (R) and I_{rms} current flowing in the circuit is $I^2_{rms}R$.

The power is to be transmitted over the large distances at high alternating voltages, so current flowing through the wires will be low because of given power (P).

For a given power level, we find that

$$P = E_{rms}I_{rms} \ (I_{rms} \text{ is low when } E_{rms} \text{ is high})$$

$$\text{Power loss} = I^2_{rms}\, R = \text{low} \qquad (\because I_{rms} \text{ is low})$$

Now at the receiving end high voltage is reduced by using step-down transformers.

Question 11. For an L-C-R circuit, the power transferred from the driving source to the driven oscillator is P = I^2 Z cos Φ.
(a) Here, the power factor cos Φ > 0, P > 0
(b) The driving force can give no energy to the oscillator (P = 0) in some cases
(c) The driving force cannot syphon out (P < 0) the energy out of oscillator
(d) The driving force can take away energy out of the oscillator
Solution:

Key concept: *Power Factor*:

(1) It may be defined as cosine of the angle of lag or lead (*i.e.*, $\cos\phi$).

(2) It is also defined as the ratio of resistance and impedance $\left(i.e., \dfrac{R}{Z}\right)$.

(3) The ratio $= \dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{W}{VA} = \dfrac{kW}{kVA} = \cos\phi$

In the given problem power transferred,

$$P = I^2 Z \cos\phi$$

where I is the current, Z = Impedance and $\cos\phi$ is power factor

(a) As power factor, $\cos\phi = \dfrac{R}{Z}$

where $\quad R > 0$ and $Z > 0$

$\Rightarrow \quad \cos\phi > 0 \Rightarrow P > 0$

(b) When $\phi = \dfrac{\pi}{2}$ (in case of L or C), $P = 0$.

(c) From (a), it is clear that $P < 0$ is not possible

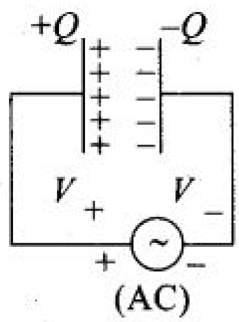
Question 12. When an AC voltage of 220 V is applied to the capacitor C
(a) the maximum voltage between plates is 220 V
(b) the current is in phase with the applied voltage

(c) the charge on the plates is in phase with the applied voltage
(d) the power delivered to the capacitor is zero
Solution: (c, d) If the alternating voltage is applied to the capacitor, the plate connected to the positive terminal of the source will be at higher potential and the plate connected to the negative terminal of source will be at lower potential. So the plates capacitor is charged.

(c) If $V = V_0 \sin \omega t$, $Q = C V_0 \sin\omega t$ or we can say that Q and emf are in phase

(d) As $P = V_{rms} I_{rms} \cos\phi$ and in case of a capacitor, $\phi = \frac{\pi}{2}$

$P = 0$, or we can say that power delivered to the capacitor is zero.



$\Rightarrow \quad P_{av} = \text{Power delivered} = 0$

Question 13.
The line that draws power supply to your house from street has
(a) zero average current
(b) 220 V average voltage
(c) voltage and current out of phase by 90°
(d) voltage and current possibly differing in phase ϕ such that $|\phi| < \frac{\pi}{2}$

Solution:
(a, d) Alternating currents are used for household supplies, which are having zero average value over a cycle.

The line is having some resistance, so power factor $\cos\phi = \frac{R}{Z} \neq 0$

So, $\quad \phi \neq \pi/2 \Rightarrow \phi < \pi/2$

i.e., phase lies between 0 and $\pi/2$.

Important point: The average value of alternating quantity for one complete cycle is zero.
The average value of *ac* over half cycle ($t = 0$ to $T/2$)

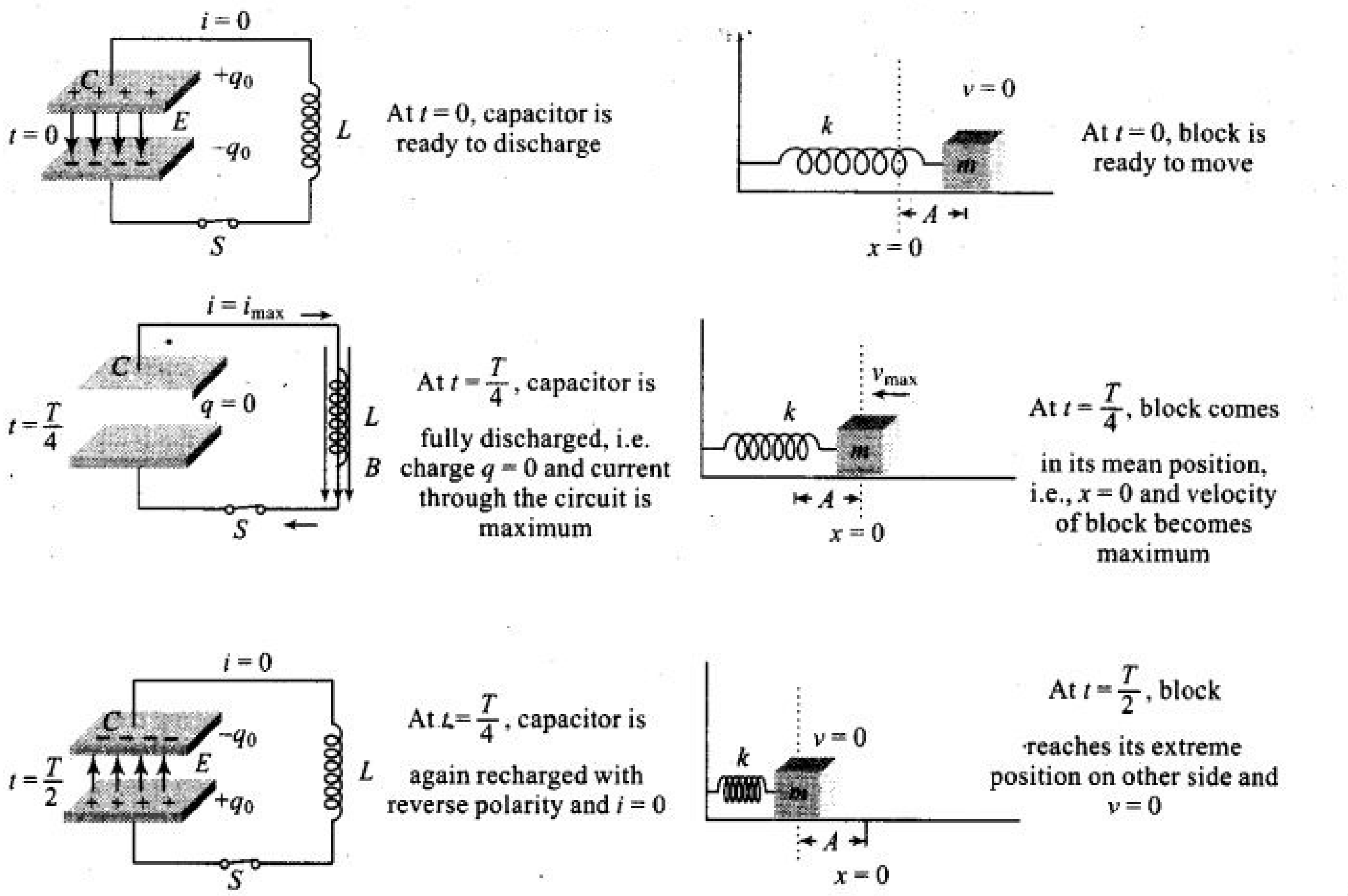
$$i_{av} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} = 0.637 i_0 = 63.7\% \text{ of } i_0.$$

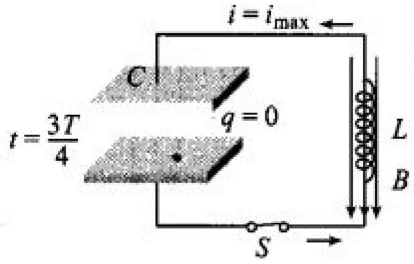
Similarly $V_{av} = \frac{2V_0}{\pi} = 0.637\, V_0 = 63.7\%$ of V_0.

Very Short Answer Type Questions
Question 14. If an L-C circuit is considered analogous to a harmonically oscillating spring-block system, which energy of the L-C circuit would be analogous to potential energy and which one analogous to kinetic energy?
Solution: When a charged capacitor C having an initial charge q0 is discharged through an inductance L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. We also assume an idealized

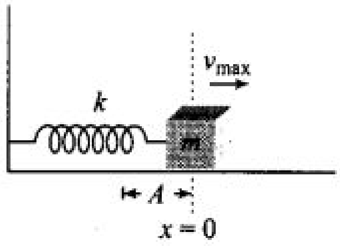
situation in which energy is not radiated away from the circuit. The total energy associated with the circuit is constant.
The oscillation of the LC circuit are an electromagnetic analog to the mechanical oscillation of a block-spring system.
The total energy of the system remains conserved.

$$\frac{1}{2}CV^2+\frac{1}{2}LI^2=\text{constant}=\frac{1}{2}CV_0^2=\frac{1}{2}LI_0^2$$

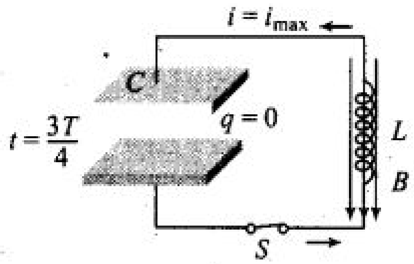




At $t = \frac{3T}{4}$, capacitor again discharges completely $i = i_{max}$



At $t = \frac{3T}{2}$, block again reaches its mean position and its velocity becomes maximum



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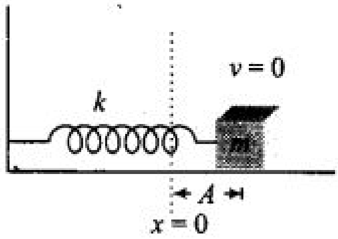


At $t = \frac{3T}{2}$, block again reaches its mean position and its velocity becomes maximum

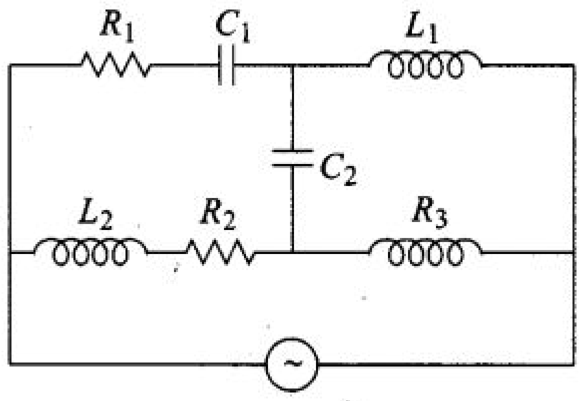
- **Comparison of oscillation of a mass spring system and an LC circuit**

Mass spring system	**v/s**	**LC circuit**
Displacement (x)		Charge (q)
Velocity (v)		Current (i)
Acceleration (a)`		Rate of change of current $\left(\frac{di}{dt}\right)$
Mass (m) [Inertia]		Inductance (L) [Inertia of electricity]
Momentum ($p = mv$)		Magnetic flux ($\phi = Li$)
Retarding force $\left(-m\frac{dv}{dt}\right)$		Self induced emf $\left(-L\frac{di}{dt}\right)$

Equation of free oscillations: $\frac{d^2x}{dt^2} = -\omega^2 x$; where $\omega = \sqrt{\frac{K}{m}}$		Equation of free oscillations: $\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right) \cdot q$; where $\omega^2 = \frac{1}{LC}$ $\Rightarrow \omega = \frac{1}{\sqrt{LC}}$
Force constant K		Capacitance C
Kinetic energy $= \frac{1}{2}mv^2$		Magnetic energy $= \frac{1}{2}Li^2$
Elastic potential energy $= \frac{1}{2}Kx^2$		Electrical potential energy $= \frac{1}{2}\frac{q^2}{C}$

If we consider an L-C circuit analogous to a harmonically oscillating spring block system. The electrostatic energy $\frac{1}{2}CV^2$ is analogous to potential energy and energy associated with moving charges (current) that is magnetic energy $\left(\frac{1}{2}LI^2\right)$ is analogous to kinetic energy.

Question 15. Draw the effective equivalent circuit of the circuit shown in figure, at very high frequencies and find the effective impedance.



Solution:
Key concept: The element with infinite resistance will be considered as open circuit and the

element with zero resistance will be considered as short circuited.

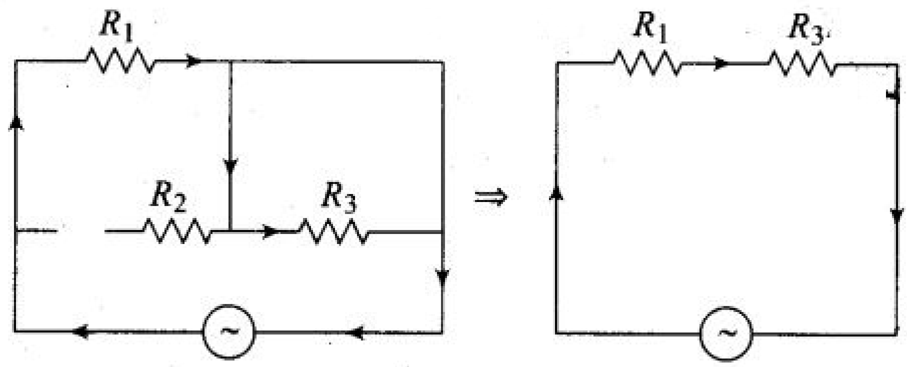
Inductive reactance $X_L = \omega L = 2\pi fL$

and capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

At very high frequencies. $X_L \to$ very large and $X_C \to$ very small

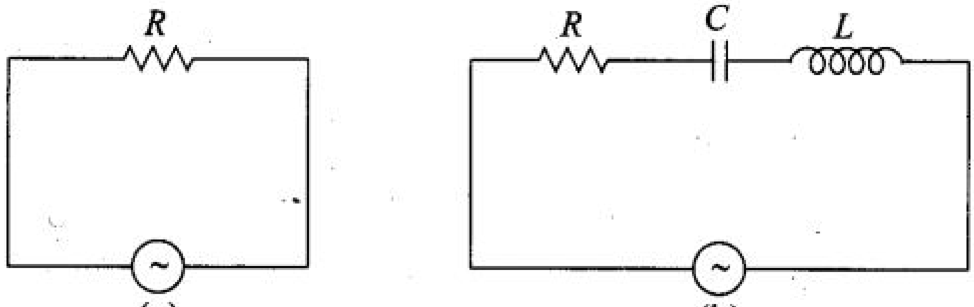
When reactance of a circuit is infinite it will be considered as open circuit. When reactance of a circuit is zero it will be considered as short circuited.

So, C_1, C_2 $\to$ shorted (if the capacitors are shorted they behave like an ordinary wire or we can simply replace them by a wire) and $L_1, L_2 \to$ opened. Then R_1 and R_3 become in series.



So, effective impedance $= Z_{eq} = R_1 + R_3$

Question 16. Study the circuits (a) and (b) shown in figure and answer the following questions.

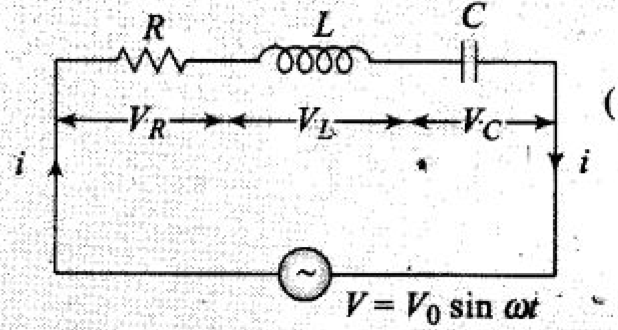


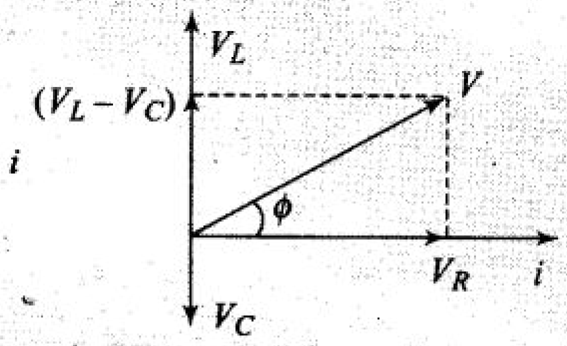
(a)

(b)

(a) Under which conditions would the rms currents in the two circuits be the same?
(b) Can the rms current in circuit (b) be larger than that in (a)?
Solution:

Key concept: Series *RLC*-Circuit





$V_R = iR$, $V_L = iX_L$, $V_C = iX_C$

Phasor diagram

(1) *Equation of current*: $i = i_0 \sin(\omega t \pm \phi)$; where $i_0 = \dfrac{V_0}{Z}$

(2) *Equation of voltage*: From phasor diagram

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

(3) *Impedance of the circuit*:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(4) *Phase difference*: From phasor diagram

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \dfrac{1}{\omega C}}{R} = \frac{2\pi \nu L - \dfrac{1}{2\pi \nu C}}{R}$$

(5) *If net reactance is inductive*: Circuit behaves as *LR* circuit

(6) *If net reactance is capacitive*: Circuit behaves as *CR* circuit

(7) *If net reactance is zero*: Means $X = X_L - X_C = 0$

$\Rightarrow X_L = X_C$. This is the condition of resonance

(8) *At resonance* (Series resonant circuit)

(i) $X_L = X_C \Rightarrow Z_{\min} = R$, *i.e.*, circuit behaves as a resistive circuit

(ii) $V_L = V_C \Rightarrow V = V_R$, *i.e.* whole applied voltage appeared across the resistance

(iii) Phase difference: $\phi = 0° \Rightarrow$ p.f. $= \cos \phi = 1$

(iv) Power consumption: $P = V_{rms} i_{rms} = \dfrac{1}{2} V_0 i_0$

(v) Current in the circuit is maximum and it is $i_0 = \dfrac{V_0}{R}$

Let us first assume, rms current in circuit $A = (I_{\text{rms}})_A$

And rms current in circuit $B = (I_{\text{rms}})_B$

$$(I_{\text{rms}})_A = \frac{E_{\text{rms}}}{Z} = \frac{E_{\text{rms}}}{R}$$

$$(I_{\text{rms}})_B = \frac{E_{\text{rms}}}{Z} = \frac{E_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(a) When $(I_{\text{rms}})_A = (I_{\text{rms}})_B$

$$R = \sqrt{R^2 + (X_L - X_C)^2}$$

$\Rightarrow \quad X_L = X_C$, resonance condition

If E_{rms} in the two circuits are same, then at resonance the rms current in LCR will be same as that in R circuit (circuit A).

(b) As $Z \geq R$

$$\Rightarrow \quad \frac{(I_{\text{rms}})_B}{(I_{\text{rms}})_A} = \frac{E_{\text{rms}}/Z}{E_{\text{rms}}/R} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{R}{Z} \leq 1$$

$\Rightarrow \quad (I_{\text{rms}})a \geq (I_{\text{rms}})b$

No, $R \leq Z$. So, rms current in circuit (b) cannot be larger than that in (a).

Question 17. Can the instantaneous power output of an AC source ever be negative? Can the average power output be negative?

Solution:

Key concept: *Power in ac Circuits*: In dc circuits power is given by $P = Vi$. But in ac circuits, since there is some phase angle between voltage and current, therefore power is defined as the product of voltage and that component of the current which is in phase with the voltage.
Thus $P = V i \cos \phi$; where V and i are *r.m.s.* value of voltage and current.

(1) *Instantaneous power*: Suppose in a circuit $V = V_0 \sin \omega t$ and $i = i_0 \sin (\omega t + \phi)$ then $P_{\text{instantaneous}} = V_i = V_0 i_0 \sin \omega t \sin (\omega t + \phi)$

(2) *Average power (True power)*: The average of instantaneous power in an ac circuit over a full cycle is called average power. Its unit is watt, *i.e.*

$$P_{av} = V_{rms} i_{rms} \cos \phi = \frac{V_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos \phi = \frac{1}{2} V_0 i_0 \cos \phi = i_{rms}^2 R = \frac{V_{rms}^2 R}{Z^2}$$

To proceed the question, let us assume the applied emf in the circuit containing L, C or a combination of L, C and R.

$$E = E_0 \sin (\omega t)$$

Hence current developed is

$$I = I_0 \sin (\omega t \pm \phi)$$

(a) Instantaneous power output of the AC source

$$P = EI = (E_0 \sin \omega t) \qquad [I_0 \sin (\omega t \pm \phi)]$$

$$= E_0 I_0 \sin \omega t \sin (\omega t + \phi)$$

$$= \frac{E_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)] \qquad \text{...(i)}$$

Clearly, from Eq. (i)

When $\cos \phi < \cos (2\omega t + \phi)$

$$P < 0$$

So, yes, the instantaneous power output of an AC source can be negative.

(b) Average power $P_{av} = \dfrac{V_0}{\sqrt{2}} \dfrac{I_0}{\sqrt{2}} \cos \phi$

$$= V_{rms} I_{rms} \cos \phi \qquad \text{...(ii)}$$

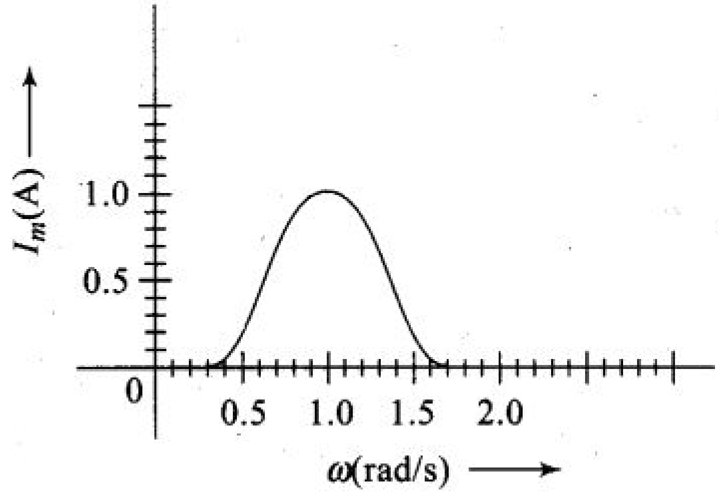
where ϕ is the phase difference.

From Eq. (ii) $\qquad P_{av} > 0$

Because $\quad \cos \phi = \dfrac{R}{Z} \geq 0$

No, the average power output of an AC source cannot be negative.

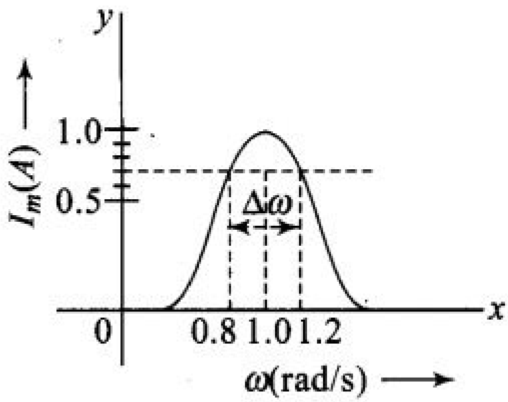
Question 18. In a series LCR circuit, the plot of I_{max} versus co is shown in figure. Find the bandwidth and mark in the figure.



Solution:

According to the given diagram,

Bandwidth $= \Delta\omega = \omega_2 - \omega_1$



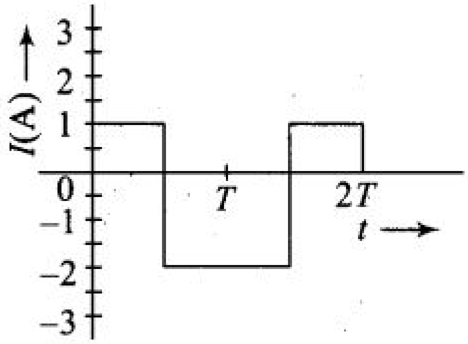
where ω_1 and ω_2 correspond to frequencies at which magnitude of current is $\frac{1}{\sqrt{2}}$ times of maximum value.

i.e., $$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1\,A}{\sqrt{2}} \approx 0.7\,A$$

From the graph these frequencies are $\omega_1 = 0.8$ rad/s and $\omega_2 = 1.2$ rad/s.

Thus, bandwidth, $\Delta\omega = \omega_1 - \omega_2 = 1.2 - 0.8 = 0.4$ rad/s

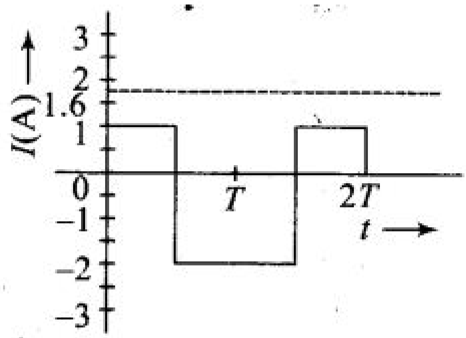
Question 19. The alternating current in a circuit is described by the graph shown in figure. Show rms current in this graph.



Solution:

According to the current versus time graph (as shown)

$$\text{rms current} = I_{\text{rms}} = \sqrt{\frac{1^2 + 2^2}{2}} = \sqrt{\frac{5}{2}} = 1.58\,\text{A} \approx 1.6\,\text{A}$$

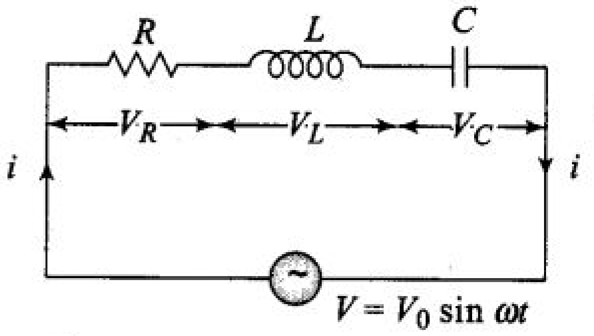


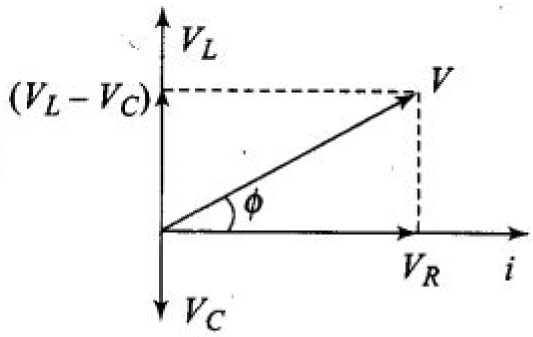
The rms value of the current (I_{rms}) = 1.6 A is indicated in the graph by dotted line.

Question 20. How does the sign of the phase angle Φ, by which the supply voltage leads the current in an L-C-R series circuit, change as the supply frequency is gradually increased from very low to very high values?

Solution:

Series LCR-Circuit





Phasor diagram

$V_R = iR$, $V_L = iX_L$, $V_C = iX_C$

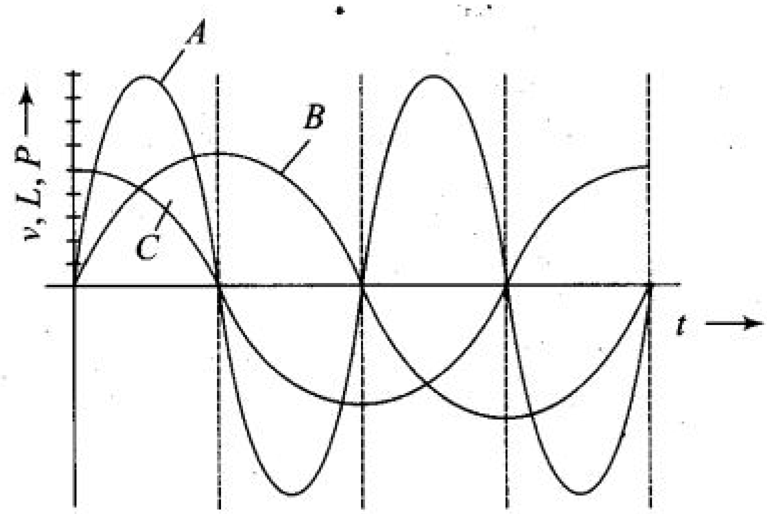
We know that the phase angle (ϕ) by which voltage leads the current in *R-L-C* series circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

At small frequencies, $X_L < X_c$,
tan is negative, i.e., $\tan\phi < 0$ (for $\nu < \nu_0$)
At resonant frequency, $X_L = X_c$,

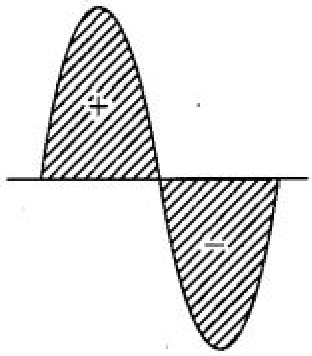
$$\therefore \qquad \tan\phi = 0\left(\text{for } \nu = \nu_0 = \frac{1}{2\pi\sqrt{2C}}\right)$$

Short Answer Type Questions
Question 21. A device 'X is connected to an AC source. The variation of voltage, current and power in one complete cycle is shown in figure.
(a) Which curve shows power consumption over a full cycle?
(b) What is the average power consumption over a cycle?
(c) Identify the device X.



Solution: (a) Power is the product of voltage and current (Power = P = VI).
So, the curve of power will be having maximum amplitude, equals to the product of amplitudes of

voltage (V) and current (I) curve. Frequencies , of B and C are-equal, therefore they represent V and I curves. So, the curve A represents power.
(b) The full cycle of the graph (as shown by shaded area in the diagram) consists of one positive and one negative symmetrical area.



Hence, average power consumption over a cycle is zero.
(c) Here phase difference between V and I is π /2 therefore, the device 'X' may be an inductor (L) or capacitor (C) or the series combination of L and C.

Question 22. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?
Solution: For a Direct Current (DC),
1 ampere = 1 coulomb/sec
Direction of AC changes with the frequency of source with the source frequency and the attractive force would average to zero. Thus, the AC ampere must be defined in terms of some property that is independent of the direction of current. Joule's heating effect is such property and hence it is used to define rms value of AC.
So, r.m.s. value of AC is equal to that value of DC, which when passed through a resistance for a given time will produce the same amount of heat as produced by the alternating current when passed through the same resistance for same time.

Question 23. A coil of 0.01 H inductance and 1 ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current.

Solution:

According to the problem, inductance $L = 0.01$ H,

resistance $R = 1\ \Omega$, voltage $(V) = 200$ V

and frequency $(f) = 50$ Hz.

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.01 = 3.14\ \Omega$$

Impedance of the *R-L* circuit

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$$

$$= \sqrt{1^2 + (2 \times 3.14 \times 50 \times 0.01)^2}$$

or $$Z = \sqrt{10.86} = 3.3\ \Omega$$

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi fL}{R} = \frac{2 \times 3.14 \times 50 \times 0.01}{1} = 3.14$$

$$\phi = \tan^{-1}(3.14) = 72^\circ$$

Phase difference $\phi = \dfrac{72 \times \pi}{180}$ rad.

Time lag between alternating voltage and current,

$$\Delta t = \frac{\phi}{\omega} = \frac{72\pi}{180 \times 2\pi \times 50} = \frac{1}{250}\text{ s} = 0.004\text{ s}$$

Question 24. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used.

Solution:

According to the problem, $P_S = 60$ W, $I_S = 0.54$ A

Let V_S, I_S and V_P, I_P are voltages and current of the secondary and primary of the transformer respectively.

Taking primary voltage as 220 V.

$\Rightarrow \quad P_S = 60\text{ W}, I_S = 0.54\text{ A}$

$\Rightarrow \quad V_S = \dfrac{60}{0.54} = 110\text{ V}$

Voltage in the secondary (E_S) is less than Voltage in the primary (E_P).

Therefore, the transformer is step down transformer.

Since, the transformation ratio

$$r = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Substituting the values, $\dfrac{110\text{ V}}{220\text{ V}} = \dfrac{I_p}{0.54\text{ A}}$

On solving $\quad I_p = 0.27$ A

Question 25. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency.

Solution: Capacitor plates get charged and discharged when an AC voltage is applied across the plates. So the current through capacitor is as a result of charging charge. Because the frequency of the capacitive circuit increases, the polarities of the charged plates change more rapidly with time, giving rise to a'larger current. The capacitance reactance (X_C) due to a capacitor C varies
as the inverse of the frequency (f) (as X_C=1/2π fC) and hence approaches zero as v approaches infinity. The current is zero in a DC capacitive circuit, which corresponds to zero proportional and infinite reactance. Also, Since X_C is inversely proportional to frequency, capacitors tend to pass high-frequency current and to block low-frequency currents and DC (just the opposite of inductors).

Question 26. Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.

Solution: The inductive reactance is given by X_L = 2πfL, X_L is proportional to the frequency and current is inversely proportional to the reactance. An inductor opposes the flow of current through it by developing a back emf according to Lenz's law. If the current is decreasing, the polarity of the induced emf will be so as to increase the current and vice-versa.
Since, the induced emf is proportional to the rate of change of current.

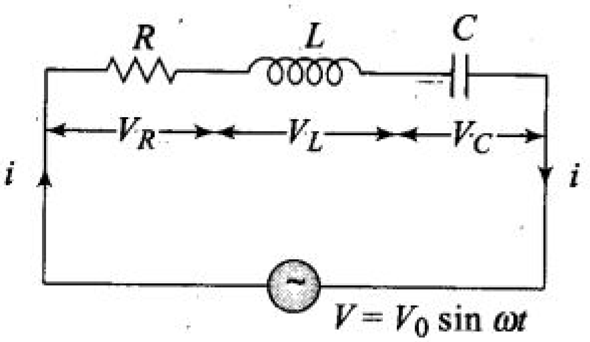
Long Answer Type Questions

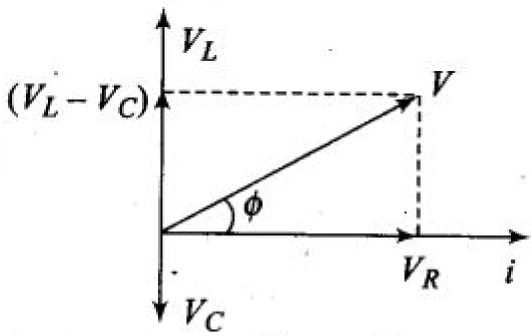
Question 27.

An electrical device draws 2 kW power from AC mains (voltage 223 V (rms) $= \sqrt{50000}$ V). The current differs (lags) in phase by $\phi\left(\tan\phi = \frac{-3}{4}\right)$ as compared to voltage. Find (a) R, (b) $X_C - X_L$ and (C) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected?

Solution:

Series LCR-Circuit





$V_R = iR$, $V_L = iX_L$, $V_C = iX_C$

Phasor diagram

$$\text{Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

According to the problem, power drawn $= P = 2$ kW $= 2000$ W

$$\tan\phi = -\frac{3}{4},$$

rms voltage, $V_{\text{rms}} = V = 223$ V

Power $\quad P = \frac{V^2}{Z}$

$\Rightarrow \quad Z = \frac{V^2}{P} = \frac{223 \times 223}{2 \times 10^3} = 25$

Impedance $Z = 25\ \Omega$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow \quad 25 = \sqrt{R^2 + (X_L - X_C)^2}$

or $\quad 625 = R^2 + (X_L - X_C)^2 \quad \ldots$(i)

Again, $\quad \tan\phi = \frac{X_C - X_L}{R} = \frac{-3}{4}$

or $\quad X_C - X_L = \frac{-3R}{4} \quad \ldots$(ii)

$$X_L - X_C = \frac{3}{4}R$$

From Eq. (ii), we put $X_L - X_C = \frac{3R}{4}$ in Eq. (i), we get

$$625 = R^2 + \left(\frac{3R}{4}\right)^2 = R^2 + \frac{9R^2}{16}$$

or $\quad 625 = \frac{25R^2}{16}$

(a) Resistance $R = \sqrt{25 \times 16} = \sqrt{400} = 20\ \Omega$

(b) $X_C - X_L = \frac{-3R}{4} = \frac{-3}{4} \times 20 = -15\ \Omega$

(c) Main current $I_M = \sqrt{2}\, I = \sqrt{2}\frac{V}{Z} = \frac{223}{25} \times \sqrt{2} = 9\sqrt{2} = 12.6$ A

Question 28.

1 MW power is to be delivered from a power station to a town 10 km away One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if

(i) power is transmitted at 220 V. Comment on the feasibility of doing this.

(ii) a step-up transformer is used to boost the voltage to 11000 V, power transmitted, then a step-down transformer is used to bring voltage to 220 V.

$(\rho_{cu} = 1.7 \times 10^{-8}$ SI unit)

Solution:

(i) The power station is 10 km away from the town.

Length of pair of Cu wires used, $L = 20$ km $= 20000$ m.

Resistance of Cu wires, $R = \rho_{cu} \dfrac{L}{A} = \rho \dfrac{L}{\pi r^2}$

$$= \frac{1.7 \times 10^{-8} \times 2 \times 10^4}{\pi (0.5 \times 10^{-2})^2} = 4\ \Omega$$

I at 220 V, $VI = 10^6$ W; $I = \dfrac{10^6}{220} = 0.45 \times 10^4$ A

$$RI^2 = \text{Power loss}$$
$$= 4 \times (0.45)^2 \times 10^8 \text{ W}$$
$$> 10^6 \text{ W}$$

Power losses are very large, hence, this method is not suitable for transmission.

(ii) When power $P = 10^6$ W is transmitted at 11000 V.

$$V'I' = 10^6 \text{ W} = 11000\, I'$$

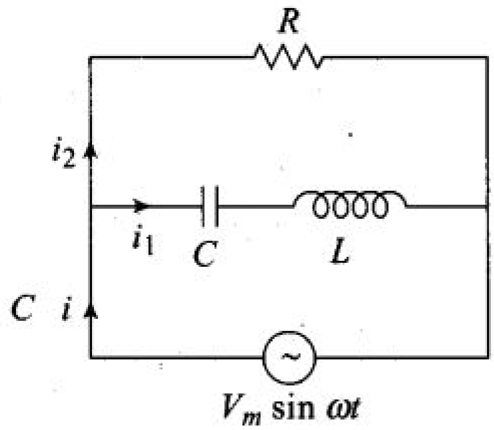
Current drawn, $I' = \dfrac{1}{1.1} \times 10^2$

Power loss $= I'^2 R = \dfrac{1}{1.21} \times 4 \times 10^4$

$$= 3.3 \times 10^4 \text{ W}$$

Fraction of power loss $= \dfrac{\Delta P}{P} = \dfrac{3.3 \times 10^4}{10^6} = 3.3\%$

Question 29. Consider the L-C-R circuit shown in figure. Find the net current i and the phase of i. Show that i = V/Z. Find the-impedance Z for this circuit.



Solution: Key concept: In the circuit given above consists of a capacitor (C) and an inductor (L) connected in series and the combination is connected in parallel with a resistance R. Due to this combination there is an oscillation of electromagnetic energy.

According to the above figure, the total current 'i' is divided into two parts i_2 through R and i_1 through a series combination of C and L.

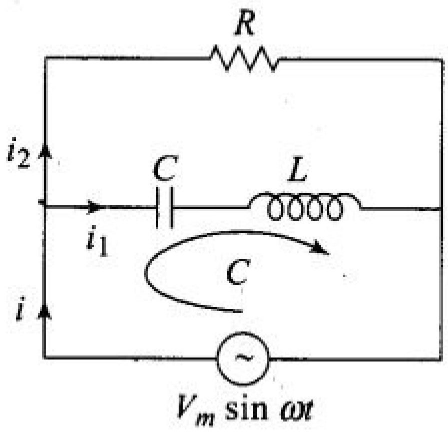
So, we get $i = i_1 + i_2$

As, $V_m \sin \omega t = R\, i_2$, [from the circuit diagram]

$$\Rightarrow \qquad i_2 = \frac{V_m \sin \omega t}{R} \qquad \ldots\text{(i)}$$

Let q_1 is charge on the capacitor at any time t, i_1 is the current in the lower circuit.

Applying KVL in the lower circuit as shown,



$$V_m \sin \omega t - \frac{q_1}{C} - \frac{L\,di_1}{dt} = 0$$

$$\Rightarrow \quad \frac{q_1}{C} + \frac{Ld^2 q_1}{dt^2} = V_m \sin \omega t \qquad \left[\because i_1 = \frac{dq_1}{dt}\right] \qquad \text{...(ii)}$$

Let $q_1 = q_m \sin(\omega t + \phi)$ ...(iii)

$$\therefore \quad i_1 = \frac{dq_1}{dt} = q_m\, \omega \cos(\omega t + \phi)$$

$$\Rightarrow \quad \frac{d(i_1)}{dt} = \frac{d^2 q_1}{dt^2} = -q_m \omega^2 \sin(\omega t + \phi)$$

Now putting these values in Eq. (ii), we get

$$q_m\left[\frac{1}{C} + L(-\omega^2)\right]\sin(\omega t + \phi) = V_m \sin \omega t$$

If $\phi = 0$ and $\left(\frac{1}{C} - L\omega^2\right) > 0$,

then $$q_m = \frac{V_m}{\left(\frac{1}{C} - L\omega^2\right)} \qquad \text{...(iv)}$$

From Eq. (iii), $i_1 = \frac{dq_1}{dt} = \omega q_m \cos(\omega t + \phi)$

Using Eq. (iv), $i_1 = \dfrac{\omega V_m \cos(\omega t + \phi)}{\dfrac{1}{C} - L\omega^2}$

Taking $\quad \phi = 0;\ i_1 = \dfrac{V_m \cos(\omega t)}{\left(\dfrac{1}{\omega C} - L\omega\right)} \qquad \ldots(\text{v})$

From Eqs. (i) and (v), we find that i_1 and i_2 are out of phase by $\dfrac{\pi}{2}$.

Now, $i_2 + i_1 = \dfrac{V_m \sin \omega t}{R} + \dfrac{V_m \cos \omega t}{\left(\dfrac{1}{\omega C} - L\omega\right)}$

Let $\dfrac{V_m}{R} = A \cos \phi$ and $\dfrac{V_m}{\left(\dfrac{1}{\omega C} - L\omega\right)} = A \sin \phi$

$$\therefore \quad \begin{aligned} i_1 + i_2 &= A \cos \phi \sin \omega t + A \sin \phi \cos \omega t \\ &= A \sin (\omega t + \phi) \end{aligned}$$

where $\quad A = \sqrt{(A\cos\phi)^2 + (A\sin\phi)^2}$

and $$\phi = \tan^{-1}\frac{B}{A}C = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)}\right]^{1/2}$$

and $$\phi = \tan^{-1}\frac{R}{\left(\frac{1}{\omega C} - L\omega\right)}$$

Hence, $$i = i_1 + i_2 = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2} \sin(\omega t + \phi)$$

or $$\frac{i}{V_m} = \frac{1}{Z} = \left[\frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2}$$

This is the expression for impedance Z of the circuit.

Important point: We should not apply the formulae of L-C-R series circuit directly in these types of problems.

Question 30.

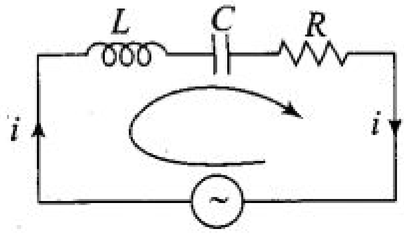
For an L-C-R circuit driven at frequency ω the equation reads

$$L\frac{di}{dt} + \frac{q}{C} + iR = V_m \sin \omega t$$

(a) Multiply the equation by i and simplify where possible.
(b) Interpret each term physically.
(c) Cast the equation in the form of a conservation of energy statement.
(d) Integrate the equation over one cycle to find that the phase difference between V and i must be acute.

Solution:

Applying KVL for the loop as shown in the figure, we can write



$$V = V_m \sin \omega t$$

$$\Rightarrow \qquad L\frac{di}{dt} + \frac{q}{C} + iR = V_m \sin \omega t \qquad \text{...(i)}$$

Multiplying both sides by i, we get

$$Li\frac{di}{dt} + \frac{q}{c}i + i^2R = (V_m i) \sin \omega t = Vi \qquad \text{...(ii)}$$

where $Li\frac{di}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$ = rate of change of energy stored in an inductor.

Now, power loss in form of heat is given i^2R.

$$\frac{q}{C}i = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = \text{rate of change of energy stored in the capacitor.}$$

Vi = rate at which driving force pours in energy. It goes into (i) ohmic loss and (ii) increase of stored energy.

Hence Eq. (ii) is in the form of conservation of energy statement. Integrating both sides of Eq. (ii) with respect to time over one full cycle ($0 \to T$) we may write

$$\int_0^T \frac{d}{dt}\left(\frac{1}{2}Li^2 + \frac{q^2}{2C}\right)dt + \int_0^T Ri^2 dt = \int_0^T Vi\,dt$$

$$\Rightarrow \qquad 0 + (+\text{ve}) = \int_0^T Vi\,dt$$

$$\Rightarrow \qquad \int_0^T Vi\,dt > 0$$ if phase difference between V and i is a constant and acute angle.

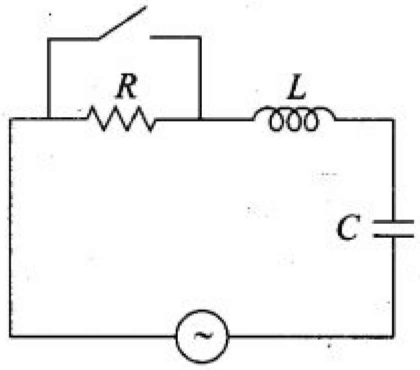
Question 31.

In the L-C-R circuit, shown in figure the AC driving voltage is $V = V_m \sin \omega t$.

(a) Write down the equation of motion for $q(t)$.

(b) At $t = t_0$, the voltage source stops and R is short circuited. Now write down how much energy is stored in each of L and C.

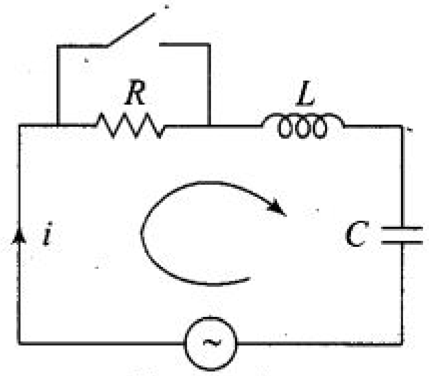
(c) Describe subsequent motion of charges.



Solution:

(a)

Key concept: We have to apply KVL in the given problem, so we have to write the equations in the form of current and charge.



$V = V_m \sin \omega t$

Given $\qquad V = V_m \sin \omega t$

Let current at any instant be i.

Applying KVL in the given circuit

$$iR + L\frac{di}{dt} + \frac{q}{C} - V_m \sin \omega t = 0 \qquad \text{...(i)}$$

Now, we can write $i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2}$

From Eq. (i), $\qquad \frac{dq}{dt}R + L\frac{d^2q}{dt^2} + \frac{q}{C} = V_m \sin \omega t$

$$\Rightarrow \quad L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin \omega t$$

This is the required equation of variation (motion) of charge.

(b) Let $q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$

$i = i_m \sin(\omega t + \phi) = q_m \omega \sin(\omega t + \phi)$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

When R is short circuited at $t = t_0$, energy is stored in L and C.

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\left[\frac{V_m}{\sqrt{(R^2 + X_C - X_L)^2}}\right]^2 \sin^2(\omega t_0 + \phi)$$

and $U_C = \frac{1}{2} \times \frac{q^2}{C} = \frac{1}{2C}\left[q^2 m \cos^2(\omega t_0 + \phi)\right]$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2$$

$$= \frac{1}{2C} \times \left(\frac{i_m}{\omega}\right)^2 \cos^2(\omega t_0 + \phi)$$

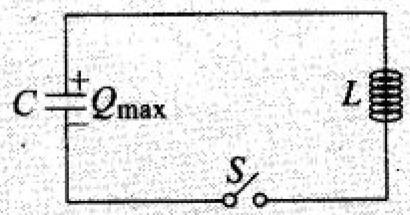
$$= \frac{i^2 m}{2C\omega^2}\cos^2(\omega t_0 + \phi) \quad [\because i_m = q_m \omega]$$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \frac{\cos^2(\omega t_0 + \phi)}{\omega^2}$$

$$= \frac{1}{2C\omega^2}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \cos^2(\omega t_0 + \phi)$$

(c) The circuit becomes an L-C oscillator when R is short circuited. The capacitor will go on discharging and all energy will go to L back and forth. Hence, there is oscillation of energy from electrostatic to magnetic and magnetic to electrostatic.

Important point: *LC Oscillation*



A capacitor is charged to a potential difference of V_0 by connecting it across a battery and then is allowed to discharge through a pure inductor of inductance L.

Initial charge on the plates of the capacitor $q_0 = CV_0$

At any instant, let the charge flown in the circuit be q and current in the circuit be i. Applying Kirchoff's law $\frac{q_0 - q}{C} - L\frac{dI}{dt} = 0$

Differentiating w.r.t. time, we get $-\frac{dq}{dt} - LC\frac{d^2I}{dt^2} = 0$

$$\frac{d^2I}{dt^2} = -\frac{1}{LC} = -\omega^2, f = \frac{1}{2\pi\sqrt{LC}}$$

The charge q on the plates of the capacitor and current I in the circuit vary sinusoidally as

$$q = q_0 \sin(\omega t + \phi) \text{ and } I = q_0\omega\cos(\omega t + \phi).$$

where ϕ is the initial phase and it depends on initial situation of the circuit of the circuit $\omega = \frac{1}{\sqrt{LC}}$

The total energy of the system remains conserved.

$$\frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \text{constant} = \frac{1}{2}CV_0^2 = \frac{1}{2}LI_0^2$$