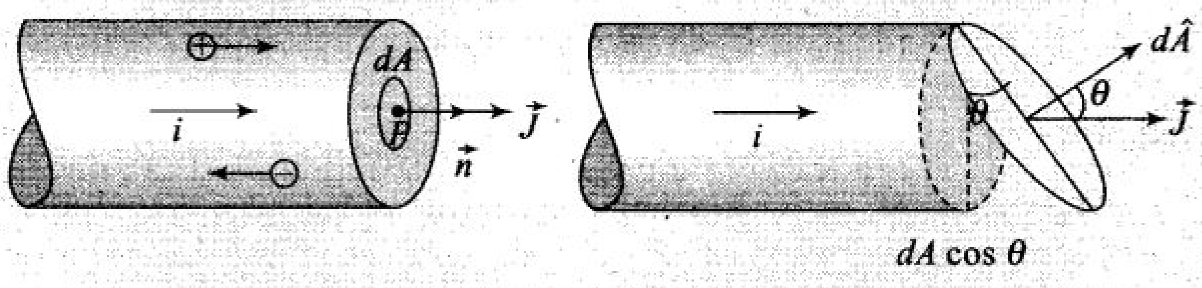
Chapter 3 - Current Electricity

Multiple Choice Questions (MCQs)

Single Correct Answer Type
Question 1. Consider a current carrying wire (current 7) in the shape of a circle. Note that as the current progresses along the wire, the direction of j (current density) changes in an exact manner, while the current I remain unaffected. The agent that is essentially responsible for is
(a) source of emf
(b) electric field produced by charges accumulated on the surface of wire
(c) the charges just behind a given segment of wire which push them just the right way by repulsion
(d) the charges ahead
Solution: (b)

Key concept: Current per unit area (taken normal to the current), I/A, is called current density and is denoted by $\vec{J}$.



The SI unit of the current density are A/m^2. The current density is also directed along E and which is also a vector quantity and the relationship is given by

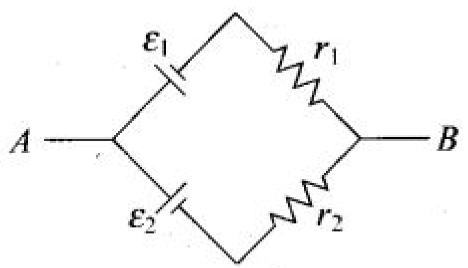
$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}} = \frac{\vec{\mathbf{E}}}{\rho}$$

where σ = conductivity and ρ = resistivity or specific resistance of the substance.

The $\vec{J}$ changes due to the electric field produced by charges accumulated on the surface of wire.

Question 2.
Two batteries of emf ε_1 and ε_2 ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.

(a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, i.e. $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$
(b) The equivalent emf ε_{eq} is smaller than ε_1
(c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always
(d) ε_{eq} is independent of internal resistances r_1 and r_2



Solution: (a) The equivalent emf of this combination is given by

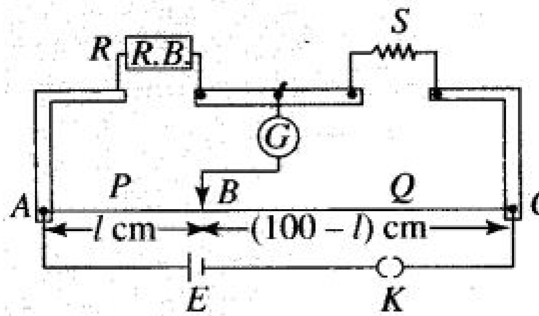
$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_1}{r_2}}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} = \frac{\varepsilon_1\left(\frac{1}{r_1} + \frac{\varepsilon_2/\varepsilon_1}{r_2}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} = \frac{\varepsilon_2\left(\frac{\varepsilon_1/\varepsilon_2}{r_1} + \frac{1}{r_2}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

As $\frac{\varepsilon_2}{\varepsilon_1} > 1 \Rightarrow \frac{\left(\frac{1}{r_1} + \frac{\varepsilon_2/\varepsilon_1}{r_2}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} > 1$ or $\varepsilon_{eq} > \varepsilon_1$ also $\frac{\varepsilon_1}{\varepsilon_2} < 1 \Rightarrow \frac{\left(\frac{\varepsilon_1/\varepsilon_2}{r_1} + \frac{1}{r_2}\right)}{\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} < 1$ or $\varepsilon_{eq} < \varepsilon_2$

Hence

$$\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2.$$

Question 3. A resistance R is to be measured using a meter bridge, a student chooses the standard resistance S to be 100 Ω He finds the null point at l_1 = 2.9 cm. He is told to attempt to improve the accuracy. Which of the following is a useful way?
(a) He should measure l_1, more accurately
(b) He should change S to 1000 Ω and repeat the experiment
(c) He should change S to 3 Ω and repeat the experiment
(d) He should have given up hope of a more accurate measurement with a meter bridge
Solution: (c)

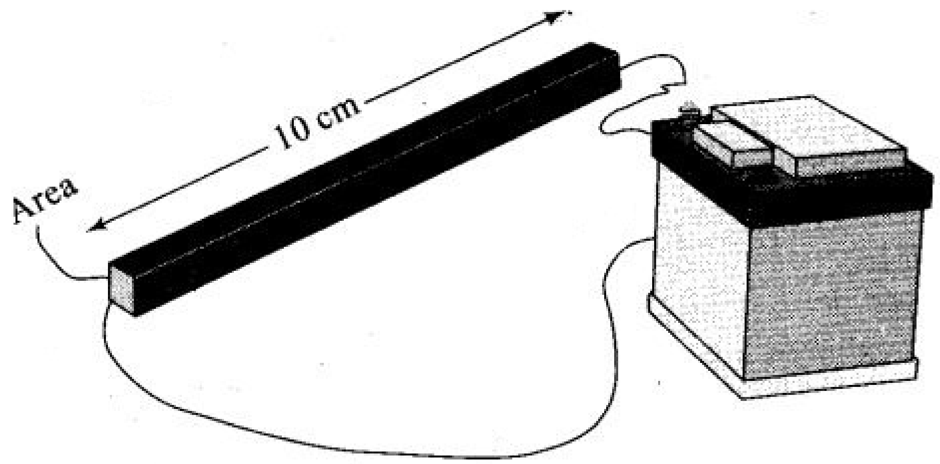


Key concept: In this problem, the concept of balanced Wheatstone bridge is to be used.
Condition of balanced wheatstone bridge: The bridge is said to be balanced if the ratio of the resistances in same branch is equal $R/S = l_1/(100-l_1)$
Wheatstone bridge is an arrangement of four resistances which can be used to measure one unknown resistance of them in terms of rest.
The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, i.e., when /, is close to 50 cm. This requires a suitable choice of S.
Since , $R/S = l_1/(100-l_1)$
Since here, R : S = 2.9 : 97.1
then the value of S is nearly 33 times to that of R. In order to make this ratio 1:1, it is necessary to reduce the value of S nearly 1/33 times, i.e., nearly 3 Ω.

Question 4. Two cells of emfs approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.
(a) The battery that runs the potentiometer should have voltage of 8 V.
(b) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V.
(c) The first portion of 50 cm of wire itself should have a potential drop of 10 V.
(d) Potentiometer is usually used for comparing resistances and not voltages.

Solution: (b)
Key concept: The potential drop along the wires of potentiometer should be greater than emfs of cells.
In a potentiometer experiment, the emf of a cell can be measured if the potential drop along the potentiometer wire is more than the emf of the cell to be determined. Here, values of emfs of two cells are given as 5 V and 10 V, therefore, the potential drop along the potentiometer wire must be more than 10 V.

Question 5. A metal rod of length 10 cm and a rectangular cross-section of 1 cm x 1/2 cm is connected to a battery across opposite faces. The resistance will be
(a) maximum when the battery is connected across 1 cm x 1/2 cm faces
(b) maximum when the battery is connected across 10 cm x 1 cm faces
(c) maximum when the battery is connected across 10 cm x 1/2 cm faces
(d) same irrespective of the three faces
Solution: (a)
Key concept: The resistance of a wire depends on various parameter, its area, material (resistivity) and length (length of the rod). Here, the metallic rod behaves as a wire.
Relationship between resistance and various parameter is given by R= ρ l/A.

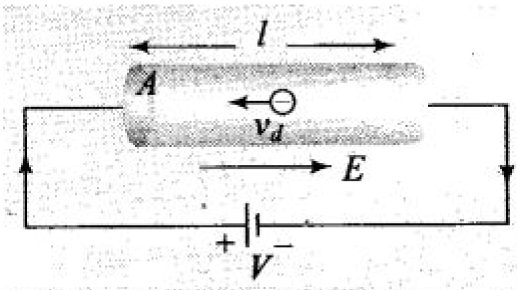


The resistance of a wire is given by

$$R = \rho \frac{l}{A}$$

For greater value of R, l must be higher and A should be lower and it is possible only when the battery is connected across 1 cm × $\left(\frac{1}{2}\right)$ cm (area of cross-section A).

Question 6. Which of the following characteristics of electrons determines the current in a conductor?
(a) Drift velocity alone
(b) Thermal velocity alone
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity
Solution: (a)
Key concept: Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for the current through it.



The direction of drift velocity for electron in a metal is opposite to that of applied electric field (i.e. current density $\vec{J}$).

$v_d \propto E$, i.e. greater the electric field, larger will be the drift velocity.

The relationship between current and drift speed is given by

$$I = neAv_d$$

Here, I is the current and v_d is the drift velocity.

So, $I \propto v_d$

Thus, only drift velocity determines the current in a conductor.

Important point: Remember direction of drift velocity and current is opposite, so we are taking the magnitude of drift velocity or drift speed of free electrons.

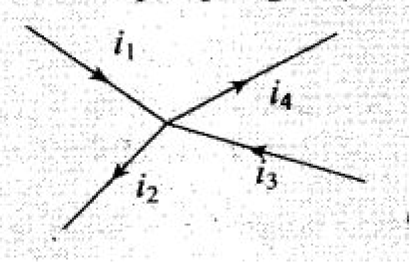
One or More Than One Correct Answer Type

Question 7. Kirchhoff's junction rule is a reflection of
(a) conservation of current density vector.
(b) conservation of charge.
(c) the fact that the momentum with which a charged particle approaches a junction is unchanged (as a vector) as the charged particle leaves the junction.
(d) the fact that there is no accumulation of charges at a junction.

Solution: (b, d)

Key concept: Junction rule: At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction.

$$i_1 + i_3 = i_2 + i_4$$



Or

Algebraic sum of the currents flowing towards any point in an electric network is zero, i.e., charges are conserved in an electric network.

The proof of this rule follows from the fact that when currents are steady, there is no accumulation of charges at any junction or at any point in a line. Thus, the total current flowing in, (which is the rate at which charge flows into the junction), must equal the total current flowing out.

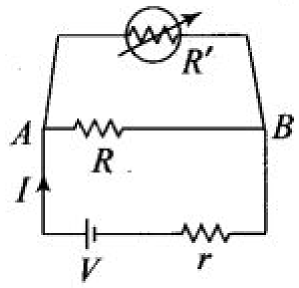
Kirchhoffs junction rule is also known as Kirchhoff's current law.

So, Kirchhoffs junction rule is the reflection of conservation of charge

Important point: Sign convention of current from a junction: We are taking outgoing current from a junction as negative. And we are taking incoming current towards a junction as positive.

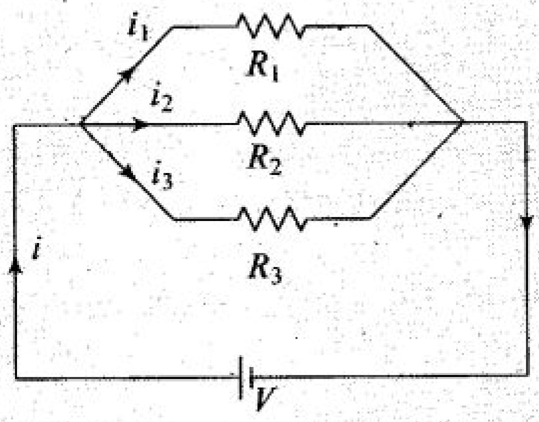
Question 8. Consider a simple circuit shown in figure stands for a variable resistance R'. R' can vary from R_0 to infinity, r is internal resistance of the battery (r <<R<<R').

(a) Potential drop across AB is nearly constant as R’ is varied.
(b) Current through R’ is nearly a constant as R’ is varied.
(c) Current I depends sensitively on R’
(d) I ≥ V/r + R always;



Solution: (a, d)

Key concept: *Parallel grouping*



Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance, i.e. $i \propto \frac{1}{R}$

In this problem, the potential drop is taking place across AB and r. Since the equivalent resistance of parallel combination of R and R' is always less than R, therefore current will be greater than or equal to $I \geq \frac{V}{r+R}$ always.

Important point: In parallel combination of resistances, the equivalent resistance is smaller than smallest resistance present in combination.

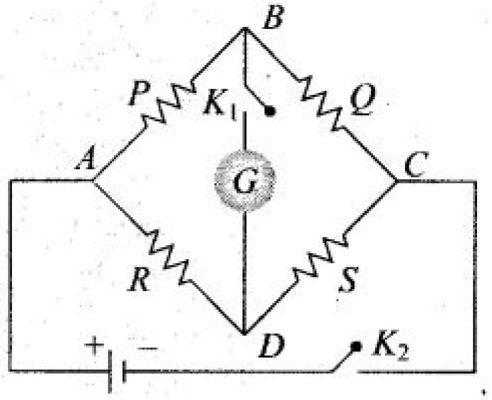
Question 9. Temperature dependence of resistivity ρ(T) of semiconductors, insulators and metals is significantly based on the following factors:
(a) number of charge carriers can change with temperature T.
(b) time interval between two successive collisions can depend on T.
(c) length of material can be a function of T.
(d) mass of carriers is a function of T.
Solution: (a, b) Resistivity is the intrinsic property of the substance.

For a metallic conductor, resistivity is given by

$$\rho = \frac{m}{ne^2\tau}$$

where n is the number of charge carriers per unit volume (number density) which can change with temperature T and τ is relaxation time (time interval between two successive collisions) which decreases with the increase of temperature $\left(T \propto \frac{1}{\tau}\right)$.

Question 10. The measurement of an unknown resistance R is to be carried out using Wheatstone bridge as given in the figure. Two students perform an experiment in two ways. The first student takes R_2 = 10 Ω and R_1 = 5 Ω. The other student takes R_2 = 1000 Ω, and R_1 = 500 Ω. In the standard arm, both take R_3 = 5 Ω.
Both find R = R_2/R_1, R_3 = 100 Ω within errors.
(a) The errors of measurement of the two students are the same
(b) Errors of measurement do depend on the accuracy with which R_2 and R_1 can be measured
(c) If the student uses large values of R_2 and R_1, the currents through the arms will be feeble. This will make determination of null point accurately more difficult.
(d) Wheatstone bridge is a very accurate instrument and has no errors of measurement
Solution: (b, c)
Key concept: Wheatstone bridge:
Wheatstone bridge is an arrangement of four resistance which can be used to measure one of them in terms of rest. Here arms AB and BC are called ratio arm and arms AC and BD are called conjugate arms.



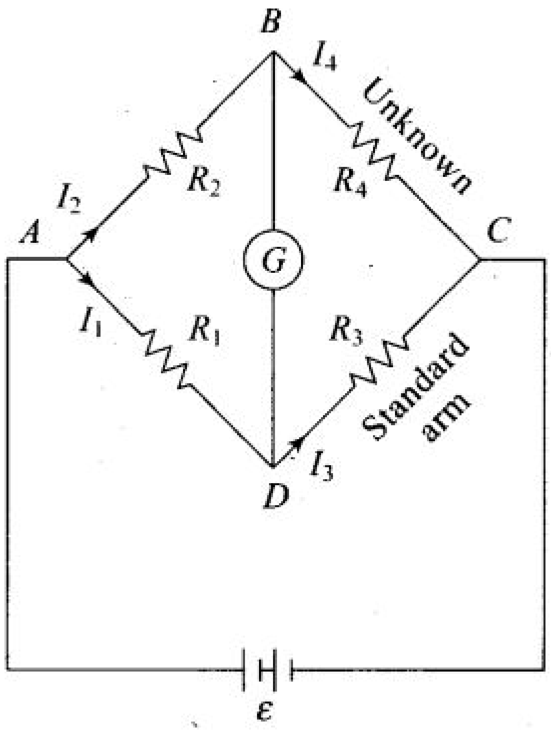
(i) *Balanced bridge:* The bridge is said to be balanced when deflection in galvanometer is zero, i.e. no current flows through the galvanometer or in other words $V_B = V_D$. In the balanced condition $\frac{P}{Q} = \frac{R}{S}$, on mutually changing the position of cell and galvanometer this condition will not change.

(ii) *Unbalanced bridge:* If the bridge is not balanced current will flow from D to B if $V_D > V_B$, i.e. $(V_A - V_D) < (V_A - V_B)$ which gives $PS > RQ$.

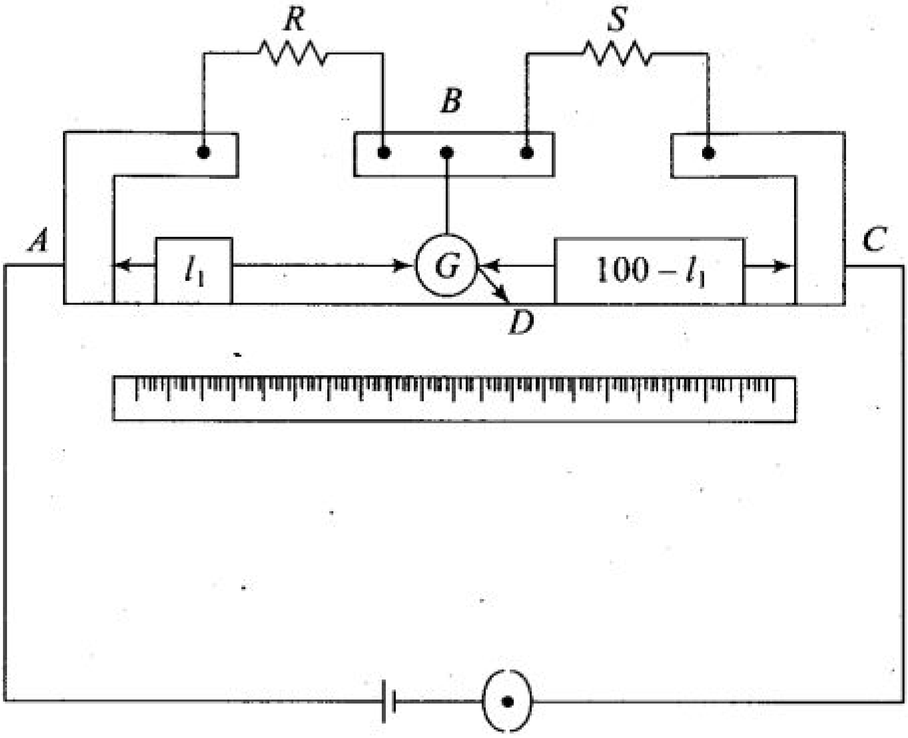
According to the problem for first student, $R_2 = 10\ \Omega$, $R_1 = 5\ \Omega$, $R_3 = 5\ \Omega$
For second student, $R_1 = 500\ \Omega$, $R_2 = 1000\ \Omega$, $R_3 = 5\ \Omega$
Let us take $R_4 = R$.
Now, according to Wheatstone bridge rule,

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \Rightarrow R_4 = R_3 \times \frac{R_2}{R_1}$$

Now putting all the values in above equation, we get R = 10 Ω for both students. Thus, we can analyse that the Wheatstone bridge is most sensitive and accurate if resistances are of same value.
Thus, the errors of measurement of the two students depend on the accuracy and sensitivity of the bridge, which in turn depends on the accuracy with which R_2 and R_1 can be measured.
The currents through the arms of bridge is very weak, when R_2 and R_1 are larger.
This can make the determination of null point accurately more difficult.



Question 11. In a meter bridge, the point D is a neutral point (figure).
(a) The meter bridge can have no other neutral. A point for this set of resistances.
(b) When the jockey contacts a point on meter wire left of D, current flows to B from the wire
(c) When the jockey contacts a point on the meter wire to the right of D, current flows from B to the wire through galvanometer
(d) When R is increased, the neutral point shifts to left



Solution: (a, c)
Key concept: Meter bridge: In case of meter bridge, the resistance wire AC is 100 cm long. Varying the position of tapping point B, bridge is balanced. If in balanced position of bridge AB = l, BC = (100 – l) so that Q/P= (100-l)/l. Also P/Q=R/S=>S=(100-l)/l R
When there is no deflection in galvanometer there is no current across the galvanometer, then points B and D are at same potential. That point at which galvanometer shows no deflection is called null point, then potential at B and neutral point D are same. When the jockey contacts a point on the meter wire to the right of D, the potential drop across AD is more than potential drop across AB, which brings the potential of point D less than that of B, hence current flows from B to D in the galvanometer wire.

Very Short Answer Type Questions
Question 12. Is the motion of a charge across junction momentum conserving? Why or why not?
Solution: In the circuit when an electron approaches a junction, in addition to the uniform E that faces it normally (which keep the drift velocity fixed), as drift velocity (v_d) is directly proportional to Electric field (E). That's why there are accumulation of charges on the surface of wires at the junction.
These produce additional electric fields. These fields alter the direction of momentum. Thus, the motion of a charge across junction is not momentum conserving.

Question 13. The relaxation time T is nearly independent of applied field E whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's law whereas the second fact leads to variation of p with temperature. Elaborate why?

Solution:

Key concept: Time interval between two successive collisions of electron with positive ions in the metallic lattice is defined as relaxation time $\tau = \frac{\text{mean free path}}{\text{r.m.s. velocity of electrons}} = \frac{\lambda}{v_{rms}}$ with rise in temperature v_{rms} increase consequently as τ decreases.

The drift velocity of the electrons is small because of the frequent collisions suffered by electrons.

Relaxation time is inversely proportional to the velocities of electrons and ions. The applied electric field produces the insignificant change in velocities of electrons at the order of 1 mm/s, whereas the change in temperature (T) affects velocities at the order of 10^2 m/s.

This decreases the relaxation time considerably in metals and consequently resistivity of metal or conductor increases as

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

Question 14. What are the advantages of the null-point method in a Wheatstone bridge? What additional measurements would be required to calculate R unknown by any other method?

Solution: In a Wheatstone bridge the main advantage of null point method is that the resistance of galvanometer does not affect the balance point, there is no need to determine current in resistances and the internal resistance of a galvanometer. It is convenient and easy method for observer.

The R unknown can calculated applying Kirchhoff's rules to the circuit. We would need additional accurate measurement of all the currents in resistances and galvanometer and internal resistance of the galvanometer.

Important point: The necessary and sufficient condition for balanced Wheatstone bridge is P/Q = R/S

where P and Q are ratio arms and R is known resistance and S is unknown resistance.

Question 15. What is the advantage of using thick metallic strips to join wires in a potentiometer?

Solution: Metallic strips have negligible resistance and need not to be counted in the length l_1, of the null point of potentiometer. That's why the thick metallic strips are used in potentiometer. It is for the convenience of experimenter as he measures only their lengths along the straight segments each of lengths 1 m.

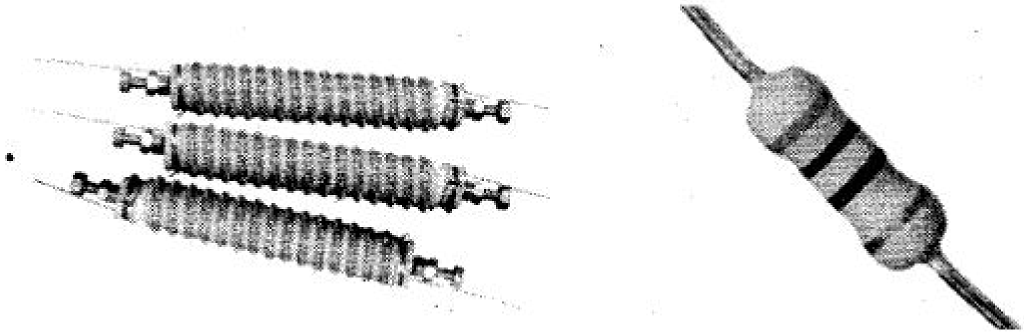
This measurements is done with the help of a centimetre scale or metre scale and leads to the accurate measurements.

Question 16. For wiring in the home, one uses Cu wires or Al wires. What considerations are involved in this?

Solution: For the selection of metal for wiring in home the main criterion are: the availability, conductivity and the cost of the metal.

The Cu wires or Al wires are used for wiring in the home. The main considerations are involved in this process are cost of metal and good conductivity of metal.

Question 17. Why are alloys used for making standard resistance coils?
Solution: Alloys are used for making standard resistance coil because they have low temperature coefficient of resistance with less temperature sensitivity.



This keeps the resistance of the wire almost constant even in small temperature change. The alloys also have high resistivity and hence high resistance, because for given length and cross-section area of conductor (L and A are constant).
R α p

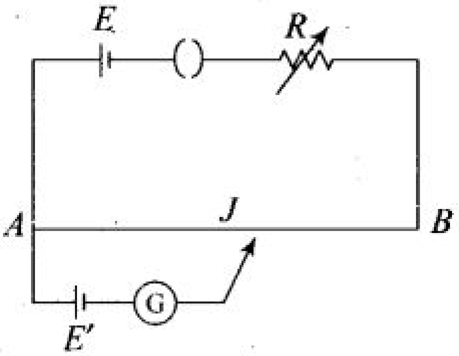
Question 18. Power P is to be delivered to a device via transmission cables having resistance R_C. If V is the voltage across R and I the current through it, find the power wasted and how can it be reduced.
Solution:

The power consumed in transmission lines is given by $P = i^2R_c$, where R_c is the resistance of connecting cables. The power is given by

$$P = VI$$

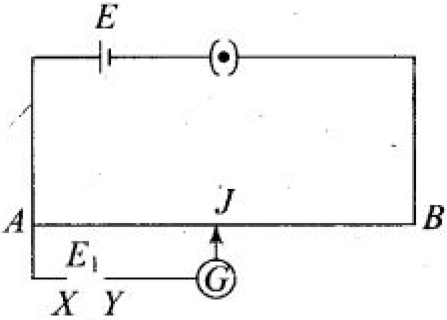
There are two ways to transmit the given power (i) at low voltage and high current or (ii) high voltage and low current. In power transmission at low voltage and high current more power is consumed as $P \propto i^2$ whereas power transmission at high voltage and low current facilitates the power transmission with minimal power consumption.
The power wastage can be reduced by transmitting power at high voltage.

Question 19. AB is a potentiometer wire (figure). If the value of R is increased, in which direction will the balance point J shift?



Solution: If the value of R is increased, the current through the wire will decrease which in turn decreases the potential difference across AB, and hence potential gradient (k) across AB decreases.
Since, at neutral point, for given emf of cell, l increases as potential gradient (k) across AB has decreased because E' = kl
Thus, with the increase of l, which will result in increase in balance length. So, jockey J will shift towards B.

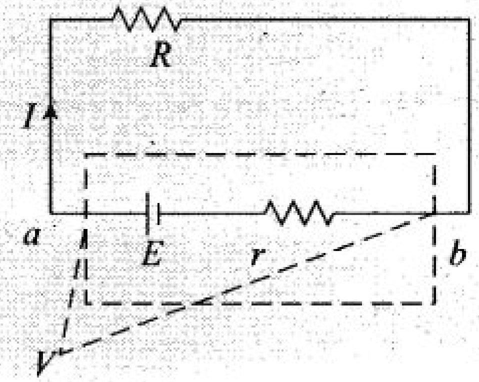
Question 20. While doing an experiment with potentiometer (figure) it was found that the deflection is one sided and (i) the deflection decreased while moving from one end A of the wire, to the end R; (ii) the deflection increased, while the jockey was moved towards the end D.
(i) Which terminal positive or negative of the cell E_1 is connected at X in case (i) and how is E_1 related to E?
(ii) Which terminal of the cell E_1 is connected at X in case (ii)?



Solution: (i) If the current in auxiliary circuit (lower circuit containing primary cell) decreases, and potential difference across A and jockey/increases. Then deflection in galvanometer is one sided and the deflection decreased, while moving from one end 'A ' of the wire to the end 'S'.
And clearly this is possible only when positive terminal of the cell E1 is connected at X and $E_1>E$.
(ii) If the current in auxiliary circuit increases, and potential difference across A and jockey J increases. Then also deflection in galvanometer is one sided.
And this is possible only when negative terminal of the cell E1 is connected at X.

Question 21. A cell of emf E and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of potential difference across R, versus R.
Solution:

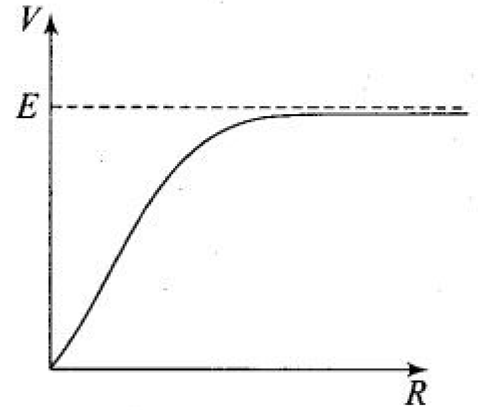
Key concept: When the cell of emf E and internal resistance r is connected across an external resistance R, the relationship between the voltage across R is given by



$$V = \frac{ER}{R+r}$$

With the increase of R, V approaches closer to E and when E is infinite, V reduces to 0.

The graphical representation of voltage across R and the resistance R is as shown.



This graph shows that potential difference increases with resistance after a certain value, it becomes constant equal to E.

Short Answer Type Questions

Question 22. First a set of n equal resistors of R each are connected in series to a battery of emf E and internal resistance R. A current / is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 10 times. What is V?

Solution: Key concept: The equivalent resistance of series combination is in series with the internal resistance R of battery and in parallel combination of resistors, the equivalent resistance of parallel combination is also in series with the internal resistance of battery.

In series combination of resistors, current I is given by $I = \dfrac{E}{R+nR}$

whereas in parallel combination current $10I$ is given by

$$\frac{E}{R+\dfrac{R}{n}} = 10I$$

Now, according to problem,

$$\frac{1+n}{1+\dfrac{1}{n}} \Rightarrow 10 = \left(\frac{1+n}{n+1}\right)n$$

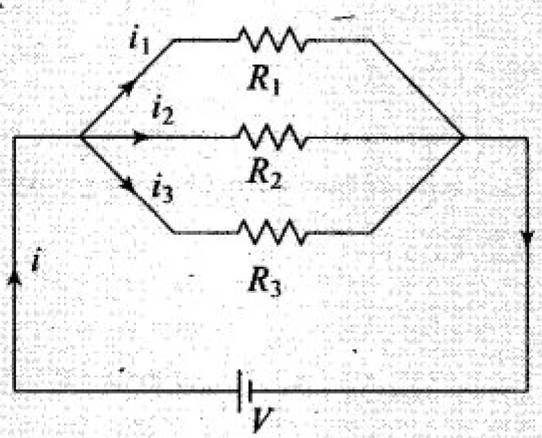
$\Rightarrow \quad n = 10$

Question 23.

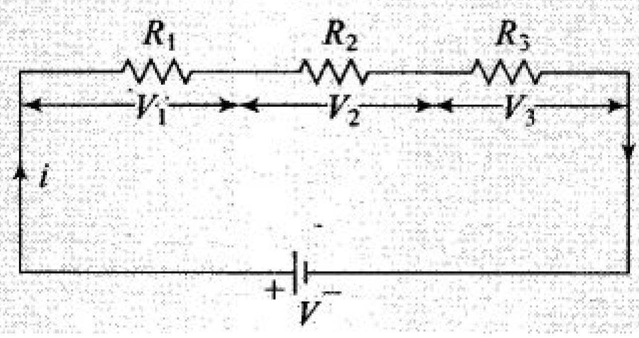
Let there be n resistors $R_1, \ldots, R_n$ with $R_{max} = \max(R_1, \ldots, R_n)$ and $R_{min} = \min\{R_1, \ldots, R_n\}$. Show that when they are connected in parallel, the resultant resistance $R_p = R_{min}$ and when they are connected in series, the resultant resistance $R_s > R_{max}$. Interpret the result physically.

Solution:

Key concept: *Parallel grouping*: Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance, i.e. $i \propto \frac{1}{R}$



Series grouping: Same current flows through each resistance but potential difference distributes in the ratio of resistance, i.e. $V \propto R$



In parallel combination: When all resistances are connected in parallel, the equivalent resistance R_p is given by

$$\frac{1}{R_p}=\frac{1}{R_1}+\ldots+\frac{1}{R_n}$$

On multiplying both sides by $R_{\min}$, we have

$$\frac{R_{\min}}{R_p}=\frac{R_{\min}}{R_1}+\frac{R_{\min}}{R_2}+\ldots+\frac{R_{\min}}{R_n}$$

Here, in RHS, there exist one term $\frac{R_{\min}}{R_{\min}}=1$ and other terms are positive, so we have

$$\frac{R_{\min}}{R_p}=\frac{R_{\min}}{R_1}+\frac{R_{\min}}{R_2}+\ldots+\frac{R_{\min}}{R_n}>1$$

This shows that the resultant resistance $R_p < R_{\min}$.

Thus, in parallel combination, the equivalent resistance of resistors even less than the minimum resistance available in combination of resistors.

In series combination: When all resistances are connected in series, the equivalent resistance R_s is given by

$$R_s = R_1 + \ldots + R_n$$

Here, in RHS, there exist one term having resistance R_{max}.
So, we have

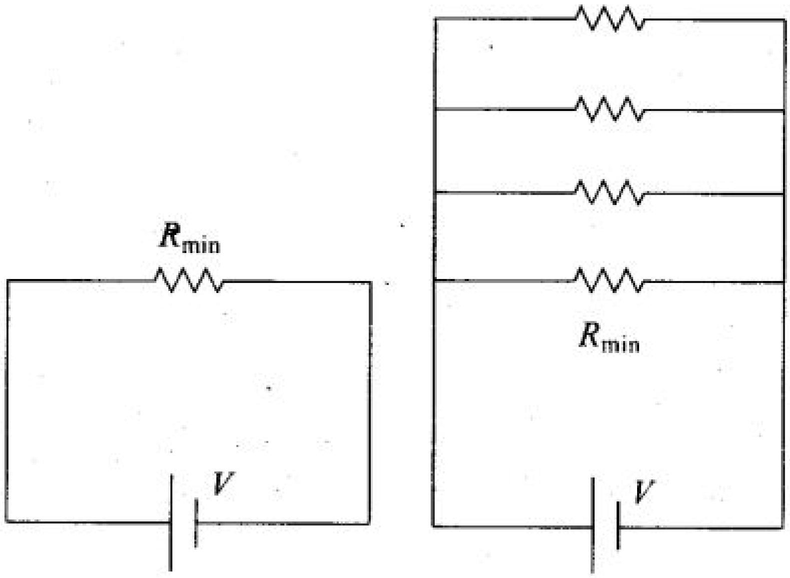
or $\quad R_s = R_1 + \ldots + R_{max} \ldots + \ldots + R_n$

$$R_s = R_1 + \ldots + R_{max} \ldots + R_n = R_{max} + \ldots (R_1 + \ldots +)R_n$$

or $\quad R_s \geq R_{max}$

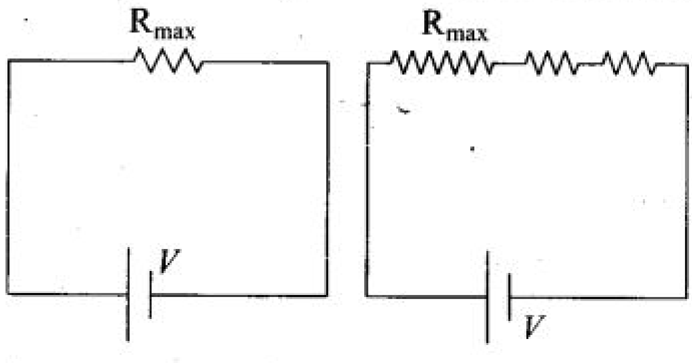
$$R_s = R_{max}(R_1 + \ldots + R_n)$$

Thus, in series combination, the equivalent resistance of resistors is greater than the maximum resistance available in combination of resistors.
Physical interpretation:



(a) (b)

In Fig. (b), R_{min} provides an equivalent route as in Fig. (a) for current. But in addition there are $(n - 1)$ routes by the remaining $(n - 1)$ resistors. Current in Fig. (b) is greater than current in Fig. (a). Effective resistance in Fig. (b) $< R_{min}$. Second circuit evidently affords a greater resistance.

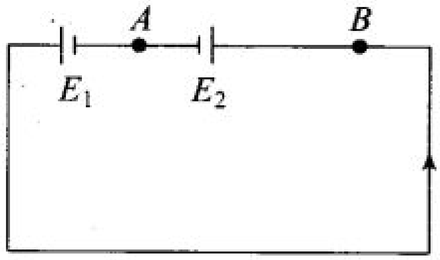


(c) (d)

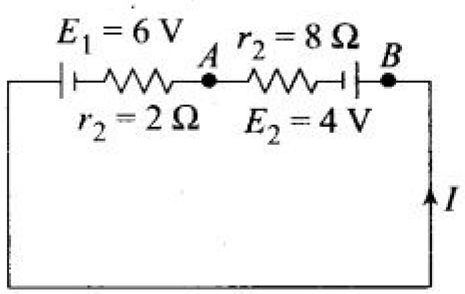
In Fig. (d), R_{max} provides an equivalent route as in Fig. (c) for current. Current in Fig. (d) < current in Fig. (c). Effective resistance in Fig. (d) $>R_{max}$. Second circuit evidently affords a greater resistance.

Question 24. The circuit in figure shows two cells connected in opposition to each other. Cell E_1 is of emf 6 V and internal resistance 2 Ω; the cell E_2 is of emf 4 V and internal

resistance 8 Ω. Find the potential difference between the points A and B.



Solution: Key concept: In this problem, after finding the electric current flow in the circuit by using Kirchoff's law or Ohm's law, the potential difference across AB can be obtained.



Applying Ohm's law.
Equivalent emf of two cells $= 6 - 4 = 2$ V and equivalent resistance $= 2\ \Omega + 8\ \Omega = 10\ \Omega$, so the electric current is given by

$$I = \frac{6-4}{2+8} = 0.2 \text{ A}$$

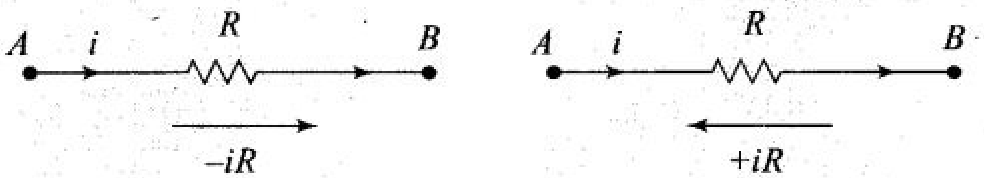
Taking loop in anti-clockwise direction, since $E_1 > E_2$
The direction of flow of current is always from high potential to low potential.
Therefore $V_B > V_A$.

$$\Rightarrow \quad V_B - 4V - (0.2) \times = V_A$$

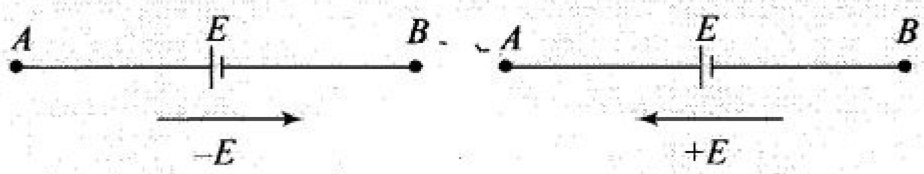
Therefore, $\quad V_B - V_A = 3.6$ V

Important point: *Sign convention for the application of Kirchoff's law:* For the application of Kirchoff's laws following sign convention are to be considered.

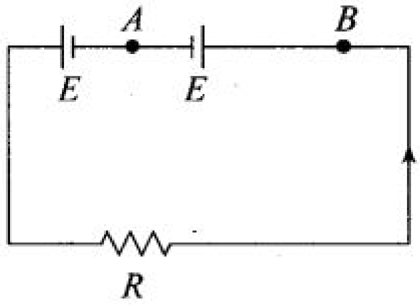
(i) The change in potential in traversing a resistance in the direction of current is $-iR$ while in the opposite direction $+iR$.



(ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction $-E$ irrespective of the direction of current in the circuit.



Question 25. Two cells of same emf E but internal resistance r_1 and r_2 are connected in series to an external resistor R (figure). What should be the value of R so that the potential difference across the terminals of the first cell becomes zero?



Solution:

In this problem first we apply Ohm's law to find current in the circuit.

Effective emf of two cells = $E + E = 2E$

Effective resistance = $R + r_1 + r_2$

So the electric current is given by

$$I = \frac{E + E}{R + r_1 + r_2}$$

The potential difference across the terminals of the first cell and putting it equal to zero.

$$V_1 = E - Ir_1 = E - \frac{2E}{r_1 + r_2 + R} r_1 = 0$$

$$E = \frac{2Er_1}{r_1 + r_2 R} \Rightarrow 1 = \frac{2r_1}{r_1 + r_2 + R}$$

or $\quad r_1 + r_2 + R = 2r_1 \Rightarrow R = r_1 - r_2$

Question 26. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistance RA to RB.

Solution:

Key concept: We know that the resistance of wire is $R=\rho\frac{l}{A}$

where A is cross-sectional area of conductor, ρ is the specific resistance or resistivity and L is the length of conductor.

The resistance of first conductor

$$R_A=\frac{\rho l}{\pi(10^{-3}\times 0.5)^2}$$

The resistance of second conductor,

$$R_B=\frac{\rho l}{\pi[(10^{-3})^2-(0.5\times 10^{-3})^2]}$$

Now, the ratio of two resistors is given by

$$\frac{R_A}{R_B}=\frac{(10^{-3})^2-(0.5\times 10^{-3})^2}{(0.5\times 10^{-3})^2}=3:1$$

Question 27. Suppose there is a circuit consisting of only resistances and batteries. Suppose one is to double (or increase it to n-times) all voltages and all. resistances. Show that currents are unaltered. Do this for circuit of Examples 3, 7 in the NCERT Text Book for Class XII.

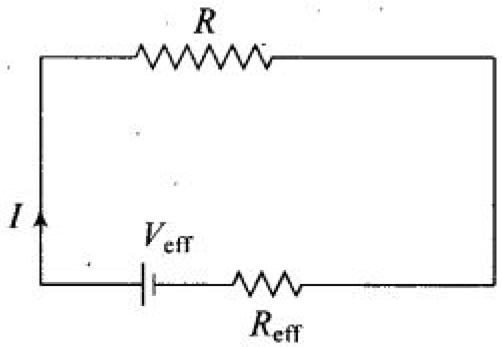
Solution:

Let us first assume the equivalent internal resistance of the battery is R_{eff}, the equivalent external resistance R and the equivalent voltage of the battery is V_{eff}.

Now by applying Ohm's law,

Then current through R is given by

$$I=\frac{V_{eff}}{R_{eff}+R}$$



Now according to the question if all the resistances and the effective voltage are increased n-times, then we have

$$V_{eff}^{new}=nV_{eff},R_{eff}^{new}=nR_{eff}$$

and $R^{new}=nR$

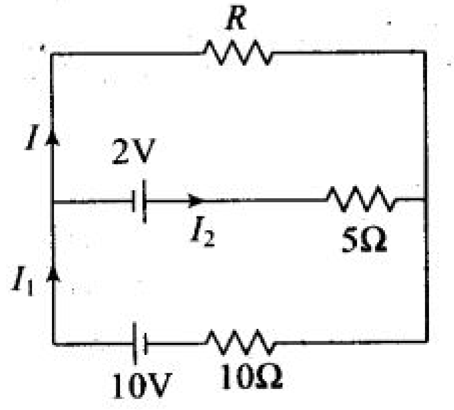
Then, the new current is given by

$$I'=\frac{nV_{eff}}{nR_{eff}+nR}=\frac{n(V_{eff})}{n(R_{eff}+R)}=\frac{(V_{eff})}{(R_{eff}+R)}=I$$

The last result of two equations is same, so we can say that current remains the same.

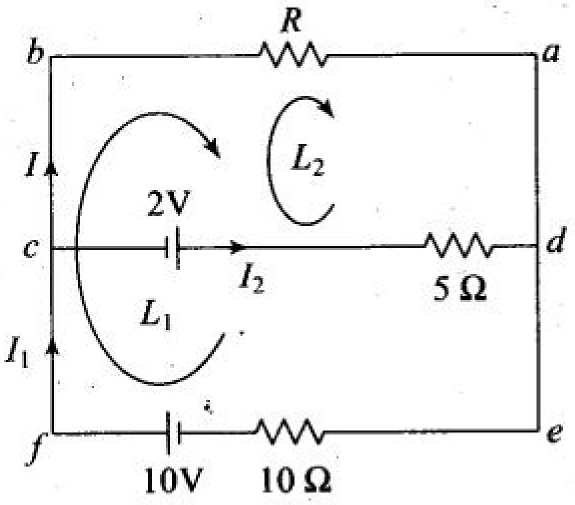
Long Answer Type Questions

Question 28. Two cells of voltage 10 V and 2 V, and internal resistances 10 Ω and 5 Ω respectively, are connected in parallel with the positive end of 10 V battery connected to negative pole of 2 V battery (figure). Find the effective voltage and effective resistance of the combination.



Solution:

In this problem first we are applying Kirchhoff's junction rule at c, $I_1 = I + I_2$



Applying Kirchhoff's voltage law in loop (e-f -b-a-e) loop L_1 outer loop, then we get

$$10 - IR - 10I_1 = 0$$

$$10 = IR + 10I_1 \qquad \ldots\text{(i)}$$

Applying Kirchhoff voltage law in loop (c-b-a-d-c) loop L_2,we get

$$-2 - IR + 5I_2 = 0$$

$$2 = 5I_2 - RI$$

As we know, $I_1 = I + I_2$ then

$$I_2 = I_1 - I$$

So the above equation can be written as

$$2 = 5(I_1 - I) - RI$$

or $\quad 4 = 10I_1 - 10I - 2RI \qquad \ldots\text{(ii)}$

Subtracting Eqs. (ii) from (i), we get

$\Rightarrow \quad 6 = 3RI + 10I$

$$2 = I\left(R + \frac{10}{3}\right)$$

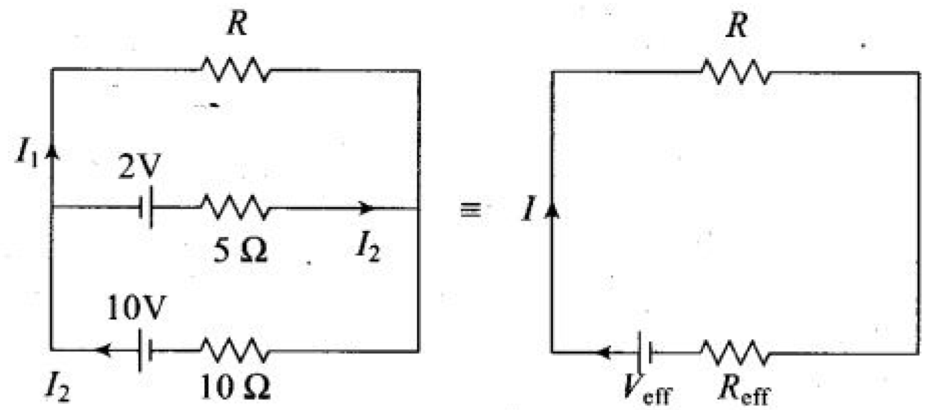
Also, the external resistance is R. The Ohm's law states that

$$V = I(R + R_{eff})$$

On comparing, we have $V = 2$ V and effective internal resistance

$$(R_{eff}) = \left(\frac{10}{3}\right)\Omega$$

Since, the equivalent internal resistance (R_{eff}) of two cells is $\left(\frac{10}{3}\right)\Omega$, being the parallel combination of 5Ω and $10\ \Omega$. *The equivalent circuit is given below:*



Question 29. A room AC runs for 5 a day at a voltage of 220 V. The wiring of the room consists of Cu of 1 mm radius and a length of 10 m. Power consumption per day is 10 commercial units. What fraction of it goes in the joule heating in wires? What would happen if the wiring is made of aluminium of the same dimensions?

$[\rho_{Cu} = 11.7 \times 10^{-8}\ \Omega m,\ \rho_{Al} = 2.7 \times 10^{-8}\ \Omega m]$

Solution:

Key concept: The energy dissipated per unit time is the power dissipated

$P = \frac{\Delta W}{\Delta t}$ and,

The power across a resistor is $P = I^2R$

Power consumption in a day, *i.e.*, in 5 = 10 units

Or power consumption per hour = 2 units

Or power consumption = 2 units = 2 kW = 2000 J/s

Also, we know that power consumption in resistor,

$$P = V \times I$$

$$\Rightarrow \quad 2000 \text{ W} = 220 \text{ V} \times I \text{ or } I \approx 9 \text{ A}$$

Now, the resistance of wire with cross-sectional area A is given by $R = \rho\frac{l}{A}$

Power consumption in first current carrying wire is given by

$$P = I^2 R$$

$$\rho\frac{l}{A}I^2 = 1.7 \times 10^{-8} \times \frac{10}{\pi \times 10^{-6}} \times 81 \text{ J/s} \approx 4 \text{ J/s}$$

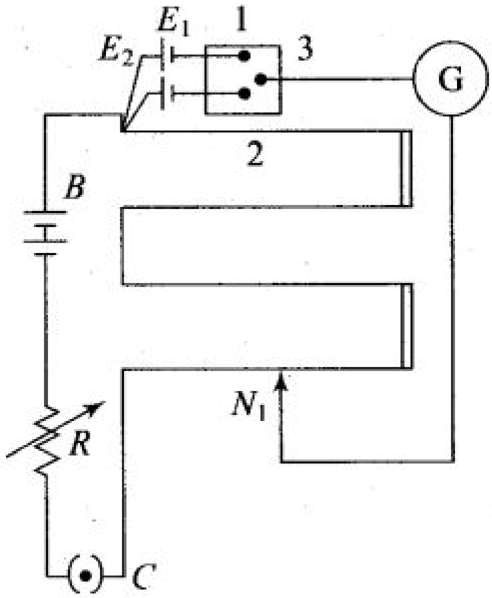
The fractional loss due to the joule heating in first wire $= \frac{4}{2000} \times 100 = 0.2\%$

Power loss in Al wire $= 4\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} = 1.6 \times 4 = 6.4$ J/s

The fractional loss due to the joule heating in second wire $= \frac{6.4}{2000} \times 100$

$= 0.32\%$

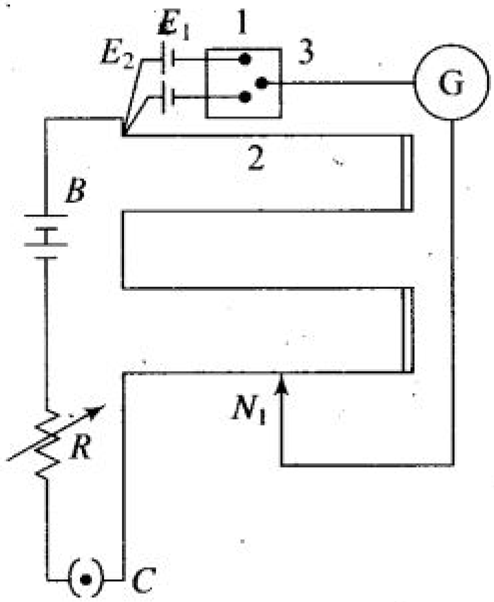
Question 30. In an experiment with a potentiometer, VB = 10 V. R is adjusted to be 50 Ω (figure). A student wanting to measure voltage E_1 of a battery (approx. 8 V) finds no null point possible. He then diminishes R to 10 Ω and is able to locate the null point on the-last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and potential drop per unit length across the wire in the second case.



Solution: Key concept: When emf of primary cell is less than the potential difference across the wires of potentiometer, only then the null point is obtained.

Equivalent resistance of potentiometer and variable resistor ($R = 50\ \Omega$) is given by $= 50\ \Omega + R'$

Equivalent voltage applied across potentiometer = 10 V



The current through the main circuit,

$$I = \frac{V}{50\Omega + R'} = \frac{10}{50\Omega + R'}$$

Potential difference across wire of potentiometer,

Since with 50 Ω resistor, null point is not obtained it is possible only when

$$\frac{10 \times R'}{50 \times R} < 8$$

$\Rightarrow \quad 10R' < 400 + 8R'$

$\cdot \quad 2R' < 400$ or $R' < 200\ \Omega$

Similarly with 10 Ω resistor, null point is obtained only when

$$\frac{10 \times R'}{10 + R'} > 8$$

$\Rightarrow \quad 2R' > 80$

$\Rightarrow \quad R' > 40$

$$\frac{10 \times \frac{3}{4} R'}{10 + R'} < 8$$

$\Rightarrow \quad 7.5R' < 80 + 8R'$

$R' > 160$

$\Rightarrow \quad 160 < R' < 200$

Any R' between 160 Ω and 200 Ω will achieve.

Since, the null point on the last (4^{th}) segment of the potentiometer, therefore potential drop across 400 cm of wire > 8 V.

This imply that potential gradient

$k \times 400\ \text{cm} > 8\ \text{V}$

or $\quad k \times 4\ \text{m} > 8\ \text{V}$

$k > 2\ \text{V/m}$

Similarly, potential drop across 300 cm wire < 8 V.

$k \times 300\ \text{cm} < 8\ \text{V}$

or $\quad k \times 3\ \text{m} < 8\ \text{V}$

$$k < 2\frac{2}{3}\ \text{V/m}$$

Thus, $\quad 2\frac{2}{3}\ \text{V/m} > k > 2\ \text{V/m}$

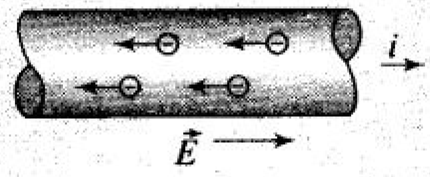
Question 31. (a) Consider circuit in figure. How much energy is absorbed by electrons from the initial state of no current (Ignore thermal motion) to the state of drift velocity?
(b) Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would number associate with energy in problem (a)? n = number of electrons/volume = $10^{29}/m^3$. Length of circuit = 10 cm cross-section . = A = $(1\ mm)^2$.
Solution:

(a)

Key concept: Relation between current and drift velocity is given by

$I = ne\,Av_d$,

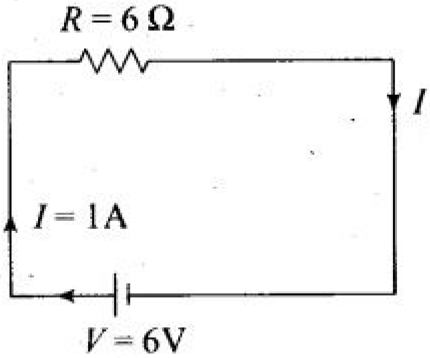
where v_d is the drift speed of electrons and n is the number density of electrons.



According to the Ohm's law current in the circuit

$$I = \frac{V}{R}$$

$$I = 6\text{ V}/6\ \Omega = 1\text{ A}$$



But, $I = neAv_d$

or $\quad v_d = \dfrac{I}{neA}$

On substituting the values,

For, n = number of electrons/volume = $10^{29}/\text{m}^3$

length of circuit = 10 cm, cross-section = $A = (1\text{ mm})^2$

$$v_d = \frac{1}{10^{29} \times 16 \times 10^{-19} \times 10^{-6}}$$

$$= \frac{1}{1.6} \times 10^{-4}\text{ m/s}$$

Therefore, the energy absorbed in the form of KE is given by
Total KE = KE of 1 electron × no. of electrons

$$\text{KE} = \frac{1}{2} m_e v_d^2 \times nAl$$

$$= \frac{1}{2} \times 9.1 \times 10^{31} \times \frac{1}{2.56} \times 10^{-8} \times 10^{29} \times 10^{-6} \times 10^{-1}$$

$$= 2 \times 10^{-17}\ \text{J}$$

(b) Ohmic loss (Power loss) is $P = I^2R = 6 \times 1^2 = 6\ \text{W} = 6\ \text{J/s}$
Since, the energy dissipated per unit time is the power dissipated.

So, $P = \dfrac{E}{t}$

Therefore, $E = P \times t$

or $\quad t = \dfrac{E}{P} = \dfrac{2 \times 10^{-17}}{6} \approx 10^{-17}\ \text{s}$

Important point: The energy dissipated per unit time is the power dissipated $P = \dfrac{\Delta W}{\Delta t}$ and,
The power across a resistor is $P = I^2R$