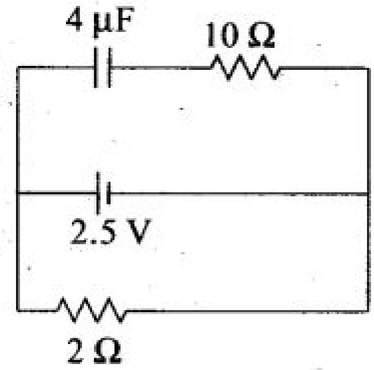
# Chapter 2 - Electrostatic Potential and Capacitance

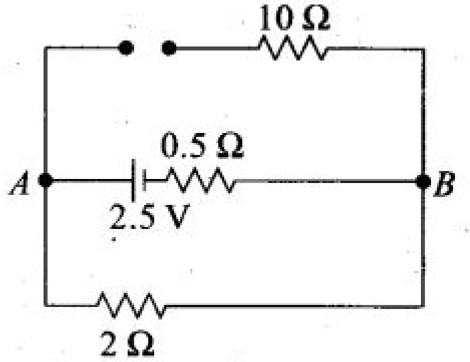
**Multiple Choice Questions**

**Single Correct Answer Type**

**Question 1.**
**A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5 Ω. The amount of charge on the capacitor plates will be**
**(a) 0 (b) 4 μC**
**(c) 16 μC (d) 8 μC**




**Solution:** (d)
Key concept: A capacitor offers zero resistance in a circuit when it is uncharged, i.e., it can be assumed as short circuited and it offers infinite resistance when it is fully charged.



At steady state

At steady state the capacitor offers infinite resistance in DC circuit and acts as open circuit as shown in figure, therefore no current flows through the capacitor and 10 Ω resistance, leaving zero potential difference across 10 Ω resistance. Hence potential difference across capacitor will be the potential difference across A and B.
The potential difference across lower and middle branch of circuit is equal to the potential difference across capacitor of upper branch of circuit.
Current flows through 2 Ω resistance from left to right, is given by I=v/R+r=1A.The potential difference across 2Ω resistance, V=IR= 1 x 2 = 2V Hence potential difference across capacitor is also 2 V.
The charge on capacitor is q = CV= (2 μF) x 2 V = 8 μC.

**Question 2. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge**
**(a) remains a constant because the electric field is uniform**
**(b) increases because the charge moves along the electric field**
**(c) decreases because the charge moves along the electric field**
**(d) decreases because the charge moves opposite to the electric field**
**Solution:** (c)
Key concept: Electric potential decreases in the direction of electric field. The direction of electric field is always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.
The positively charged particle experiences electrostatic force along the direction of electric field, hence moves in the direction of electric field. Thus, positive work is done by the electric field on the charge .We know

$$W_{\text{electrical}} = -\Delta U = -q\Delta V = q(V_{\text{initial}} - V_{\text{final}})$$

Hence electrostatic potential energy of the positive charge decreases.

**Question 3. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.**
**(a) The work done in Fig. (i) is the greatest.**
**(b) The work done in Fig. (ii) is least.**
**(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii).**
**(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in**

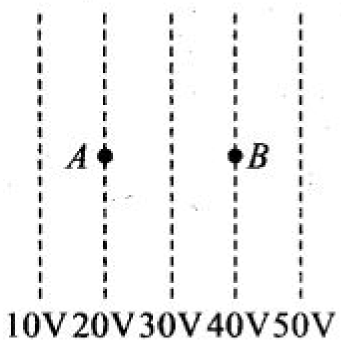


Fig. (i)

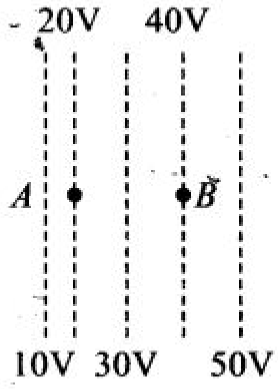


Fig. (ii)

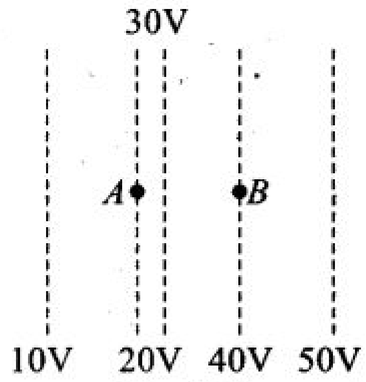
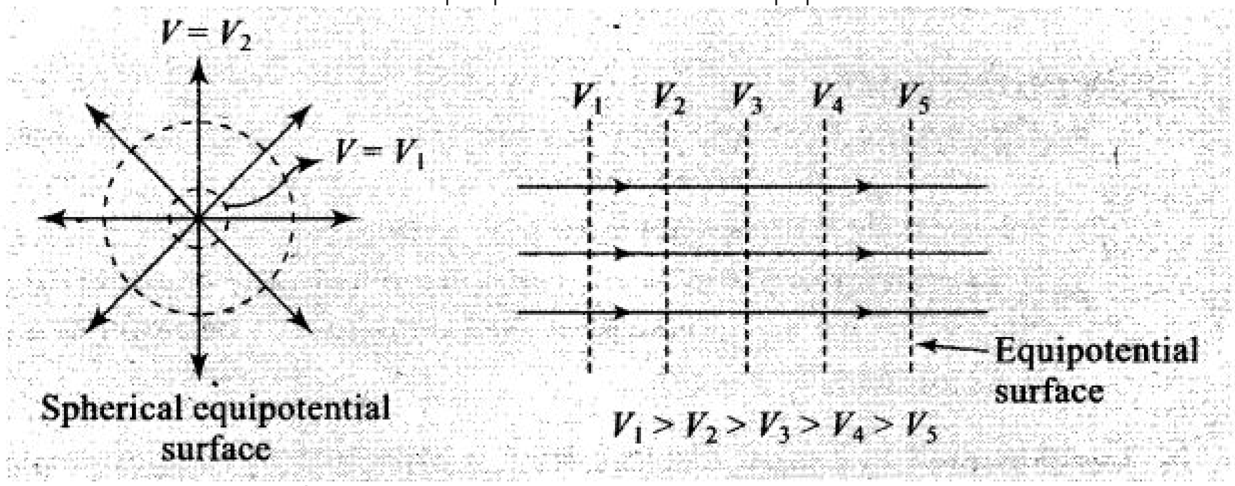


Fig. (iii)

**Solution:** (c)
Key concept: For a given charge distribution, locus of all points or regions for which the electric potential has a constant value are called equipotential regions. Such equipotential can be surfaces, volumes or lines. Regarding equipotential surface the following points should be kept in mind:
• The density of the equipotential lines gives an idea about the magnitude of electric field. Higher the density, larger the field strength.
• The direction of electric field is perpendicular to the equipotential surfaces or lines.




• The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.
• For a uniform electric field, the equipotential surfaces are a family of plane perpendicular to the field lines.
• A metallic surface ofany shape is an equipotential surface.
• Equipotential surfaces can never cross each other.
• The work done in moving a charge along an equipotential surface is always zero.
As the direction of electric field is always perpendicular to one equipotential surface maintained at high electrostatic potential than other equipotential surface maintained at low electrostatic potential. Hence direction of electric field is from B to A in all three cases.
The positively charged particle experiences electrostatic force along the direction of electric field, hence moves in the direction opposite to electric field. Thus, the work done by the electric field on the charge will be negative. We know

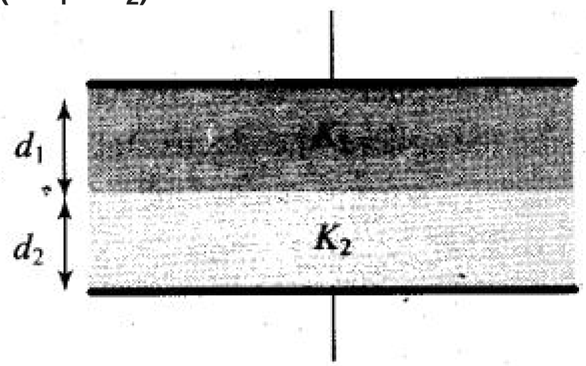
$$W_{\text{electrical}} = -\Delta U = -q\Delta V = q(V_{\text{initial}} - V_{\text{final}})$$

Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

**Question 4. The electrostatic potentiaLon the surface of a charged conducting sphere is 100 V. Two statements are made in this regard.**
**$S_1$ : At any point inside the sphere, electric intensity is zero.**
**$S_2$: At any point inside the sphere, the electrostatic potential is 100 V.**
**Which of the following is a correct statement?**
**(a) $S_1$ is true but $S_2$ is false**
**(b) Both $S_1$ and $S_2$ are false**
**(c) $S_1$ is true, $S_2$ is also true and 5, is the cause of $S_2$**
**(d) $S_2$ is true, $S_2$ is also true but the statements are independent**
**Solution:** (c) We know, the electric field intensity E and electric potential V are related
E=dV/dr
If electric field intensity E= 0, then dV/dr = 0. It means, E = 0 inside the charged conducting sphere causes uniform potential inside the sphere. Hence uniform electrostatic potential 100 V will be at any point inside the sphere.
Important points:
• The electric field zero does not necessary imply that electric potential is zero. E.g., the electric field intensity at any point inside the charged spherical shell is zero but there may exist non-zero electric potential.
• If two charged particles of same magnitude but opposite sign are
placed, the electric potential at the midpoint will be zero but electric field is not equal to zero. *

**Question 5. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately**
**(a) spheres (b) planes**
**(c) paraboloids (d) ellipsoids**
**Solution:** (a) The collection of charges, whose total sum is not zero, with regard to great distance can be considered as a single point charge. The equipotential surfaces due to a point charge are spherical.
Important point:
The electric potential due to point charge q is given by $V=q/4\pi\epsilon_0 r$
It means electric potential due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

**Question 6. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness $d_1$ and dielectric constant $K_1$ and the other has thickness $d_2$ and dielectric constant $K_2$ as shown in figure. This arrangement can be thought as a dielectric slab of thickness d (= $d_1 + d_2$) and effective dielectric constant K. Then K is**

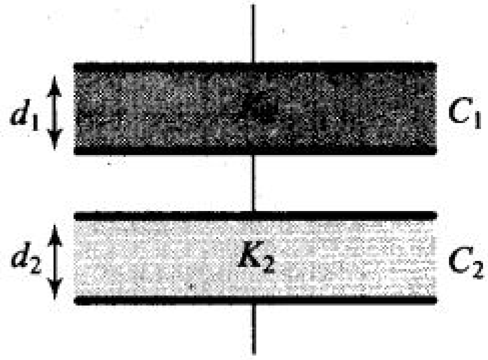



(a) $\dfrac{K_1d_1+K_2d_2}{d_1+d_2}$ (b) $\dfrac{K_1d_1+K_2d_2}{K_1+K_2}$

(c) $\dfrac{K_1K_2+(d_1+d_2)}{(K_1d_1+K_2d_2)}$ (d) $\dfrac{2K_1K_2}{K_1+K_2}$

**Solution:** (c) Here the system can be considered as two capacitors $C_1$ and $C_2$ connected in series as shown in figure.

The capacitance of parallel plate capacitor filled with dielectric block has thickness $d_1$ and dielectric constant $K_2$ is given by




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{\frac{K_1\varepsilon_0 A}{d_1} \cdot \frac{K_2\varepsilon_0 A}{d_2}}{\frac{K_1\varepsilon_0 A}{d_1} + \frac{K_2\varepsilon_0 A}{d_2}} = \frac{K_1 K_2 \varepsilon_0 A}{K_1 d_2 + K_2 d_1} \quad \ldots\text{(i)}$$

**We can write the equivalent capacitance as**

$$C = \frac{K\varepsilon_0 A}{d_1 + d_2} \quad \ldots\text{(ii)}$$

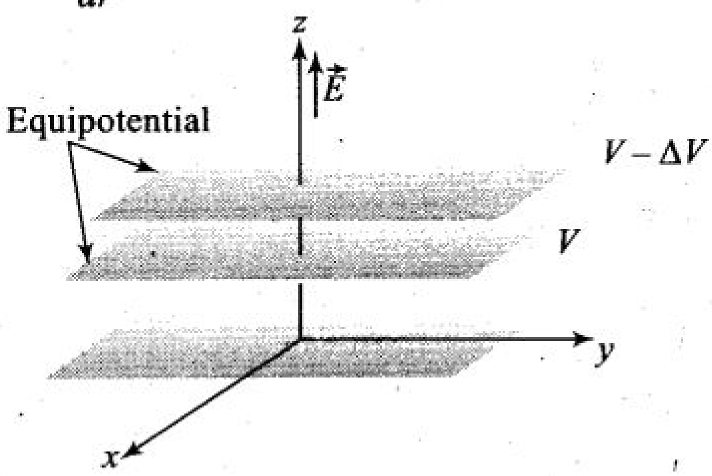
**On comparing (i) and (ii) we have**

$$K = \frac{K_1 K_2 (d_1 + d_2)}{K_1 d_2 + K_2 d_1}$$

**One or More Than One Correct Answer Type**

**Question 7. Consider a uniform electric field in the z -direction. The potential is a constant**
**(a) in all space (b) for any x for a given z**
**(c) for any y for a given z (d) on the x-y plane for a given z**
**Solution:** (b c, d) We know, the electric field intensity E and electric potential V are

$$E = -\frac{dV}{dr}$$




Electric potential decreases inf the direction of electric field. The direction of electric field is always

perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.
The electric field in z-direction suggest that equipotential surfaces are in x-y plane. Therefore the potential is a constant for any x for a given z, for any y for a given z and on the x-y plane for a given z.

**Question 8. Equipotential surfaces**
**(a) are closer in regions of large electric fields compared to regions of lower electric fields**
**(b) will be more crowded near sharp edges of a conductor**
**(c) will be more crowded near regions of large charge densities**
**(d) will always be equally spaced**
**Solution:** (a, b, c)
Key concept: The density of the equipotential lines gives an idea about the magnitude of electric field. Higher the density, larger the field strength.
We know, the electric field intensity E and electric potential V are related as

$$E = -\frac{dV}{dr}$$

Or we can write $|E| = \frac{\Delta V}{\Delta r}$

For a given $\Delta V$, $|E| \propto \frac{1}{\Delta r}$

Hence the electric field intensity E is inversely proportional to the separation between equipotential surfaces. So, equipotential surfaces are closer in regions of large electric fields.
As electric field intensities is large near sharp edges of charged conductor and near regions of large charge densities. Therefore, equipotential surfaces are closer at such places.

**Question 9. The work done to move a charge along an equipotential from A to B**

(a) cannot be defined as $-\int_A^B E \cdot dl$

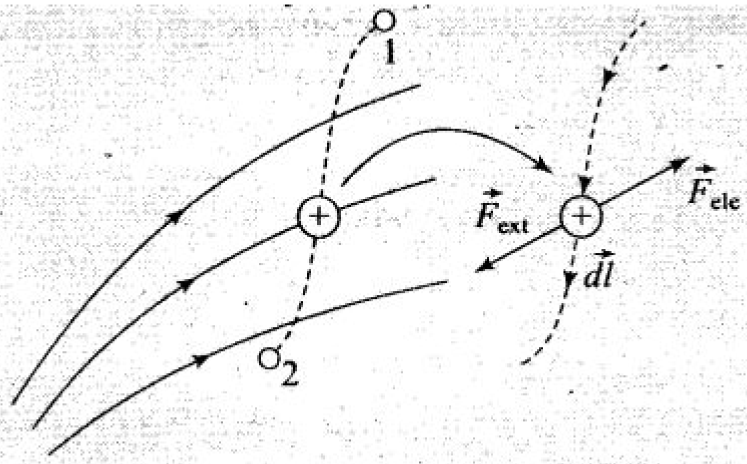
(b) must be defined as $-\int_A^B E \cdot dl$

(c) is zero

(d) can have a non-zero value

**Solution:** (b, c)
Key concept: The work done by the external agent in shifting the test charge along the dashed

line from 1 to 2 is




The external agent does a work $W = -q\int_1^2 \vec{E}\cdot d\vec{l}$ in transporting the test charge $q$ slowly from the positions 1 to 2 in the static electric field.

$$W_{ext} = \int_1^2 \vec{F}_{ext}\cdot \vec{dl} = \int_1^2 (-q\vec{E})\cdot \vec{dl} = -q\int_1^2 \vec{E}\cdot \vec{dl}$$

We know $V_A - V_B = -\int_A^B \vec{E}\cdot \vec{dl}$ ...(i)

$W_{electrical} = -\Delta U = -q\Delta V = q(V_A - V_B)$ ...(ii)

Hence from (i) and (ii), $W_{electrical} = q(V_A - V_B) = -q\int_A^B \vec{E}\cdot \vec{dl}$

If we want to calculate the work done to move a charge along an equipotential from $A$ to $B$,

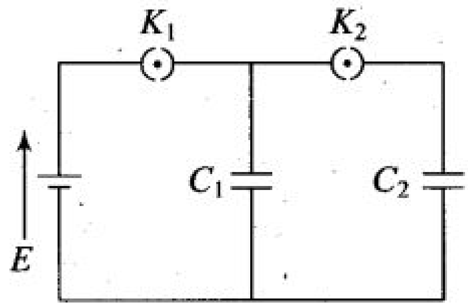
For equipotential surface $V_A = V_B$ hence $W = 0$.

Also electric field is perpendicular to equipotential surface, hence $\vec{E}\cdot \vec{dl} \Rightarrow W_{electrical} = 0$

**Question 10. In a region of constant potential**
**(a) the electric field is uniform**
**(b) the electric field is zero**
**(c) there can be no charge inside the region**
**(d) the electric field shall necessarily change if a charge is placed outside the region**

**Solution:** (b, c) We know, the electric field intensity E and electric potential V are dV related as E =- dV/dr
or we can write |E|=ΔV/Δr
The electric field intensity E and electric potential V are related as E = 0 and for V = constant,dV/dr=0 this imply that electric field intensity E = 0.
If some charge is present inside the region then electric field cannot be zero at that region, for this V = constant is not valid.

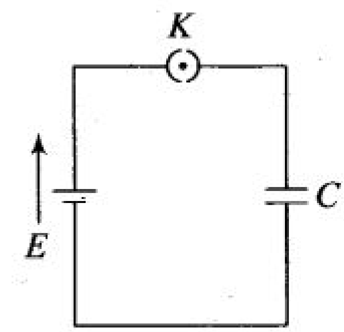
**Question 11.**




**In the circuit shown in figure initially key $K_1$ is closed and key $K_2$ is open. Then $K_1$ is opened and $K_2$ is closed (order is important).**
**[Take $Q'_1$ and $Q'_2$ as charges on $C_1$ and $C_2$ and $V_1$ and $V_2$ as voltage respectively.]**
**Then, E**
**(a) charge on C, gets redistributed such that $V_1 = V_2$**
**(b) charge on $C_1$ gets redistributed such that $Q'_1 = Q'_2$**
**(c) charge on $C_1$ gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$**
**(d) charge on $C_1$ gets redistributed such that $Q'_1 + Q'_2=Q$**
**Solution:** (a, d) Initially key $K_1$ is closed and key $K_2$ is open, the capacitor $C_1$ is charged by battery and capacitor $C_2$ is still uncharged. Now $K_1$ is opened and $K_2$ is closed, the capacitors $C_1$ and $C_2$ both are connected in parallel. The charge stored by capacitor $C_1$, gets redistributed between $C_1$ and $C_2$ till their potentials become same, i.e., $V_2 = V_1$.
By law of conservation of charge, the charge stored in capacitor Cx is equal to sum of charges on capacitors $C_1$ and $C_2$ when $K_1$ is opened and $K_2$ is closed, i.e.,
$Q'_1+Q'_2=Q$

**Question 12. If a conductor has a potential $V\neq0$ and there are no charges anywhere else outside, then**
**(a) there must be charges on the surface or inside itself**
**(b) there cannot be any charge in the body of the conductor**
**(c) there must be charges only on the surface**
**(d) there must be charges inside the surface**
**Solution:** (a, b) The potential of a body is due to charge of the body and due to the charge of surrounding. If tfiere are no charges anywhere else outside, then the potential of the body will be due to its own charge. If there is a cavity inside a conducting body, then charge can be placed inside the body. Hence there must be charges on its surface or inside itself. Hence option (a) is correct. The charge resides on the outer surface of a closed charged conductor. Hence there cannot be any charge in the body of the conductor. Hence option (b) is correct.

**Question 13.**




**A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations.**
**A. Key K is kept closed and plates of capacitors are moved apart using insulating handle.**
**B. Key K is opened and plates of capacitors are moved apart using insulating handle.**
**Choose the correct option(s).**
**(a) In A, Q remains the same but G changes**
**(b) In B, V remains the same but C changes**
**(c) In A, V remains the same hence Q changes**
**(d) In B ,Q remains the same hence V changes**
**Solution:** (c, d) The battery maintains the potential difference across connected capacitor in every circumstance. However, charge stored by disconnected charged capacitor remains conserved.
**Case A:** When key K is kept closed and plates of capacitors are moved apart using insulating handle.
The battery maintains the potential difference across connected capacitor in every circumstance. The separation between two plates increases which in turn decreases its capacitance ($C=\epsilon_0A/d$)and potential difference across
connected capacitor continue to be the same as capacitor is still connected with battery. Hence,

the charge stored decreases as Q = CV.
**Case B:** When key K is opened and plates of capacitors are moved apart using insulating handle. The charge stored by isolated charged capacitor remains conserved. The separation between two plates is increasing which in turn decreases its capacitance with the decrease of capacitance, potential difference V increases as V=Q/C.

**Very Short Answer Type Questions**

**Question 14. Consider two conducting spheres of radii $R_1$ and $R_2$ with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.**
**Solution:** Since, the two spheres are at the same potential, therefore

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1}=\frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2}\Rightarrow\frac{R_1}{\varepsilon_0}\frac{q}{4\pi R_1^2}=\frac{R_2}{\varepsilon_0}\frac{q_2}{4\pi R_2^2}$$

or $$\sigma_1 R_1=\sigma_2 R_2\Rightarrow\frac{\sigma_1}{\sigma_2}=\frac{R_2}{R_1}$$

As $R_2 > R_1$, this imply that $\sigma_1 > \sigma_2$.
The charge density of the smaller sphere is more than that of the larger one.

**Question 15. Do free electrons travel to region of higher potential or lower potential?**
**Solution:** The force on a charge particle in electric field F = qE
The free electrons (negative charge) experience electrostatic force in a direction opposite to the direction of electric field.
Thedirection of electric field is always from higher potential tolower. Hence direction of travel of electrons is from lower potential to region of higher potential.

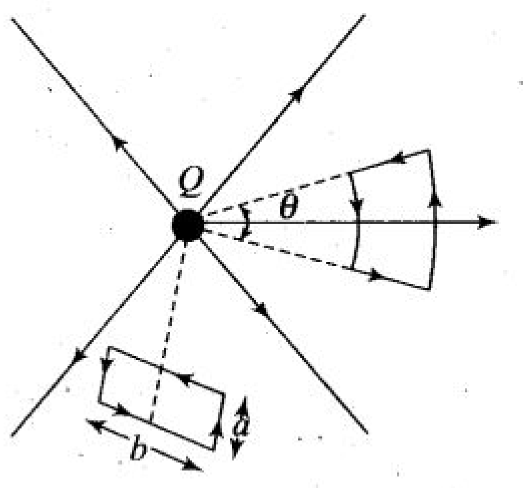
**Question 16. Can there be a potential difference between two adjacent conductors carrying the same charge?**
**Solution:** Yes, if the sizes are different.
Explanation: We define capacitance of a conductor C = Q/V is the charge of conductor and V is the potential of the conductor. For given charge potential $V \propto 1/C$. The capacity of conductor depends on its geometry, so two adjacent conductors carrying the same charge of different dimensions may have different potentials.

**Question 17. Can the potential function have a maximum or minimum in free space?**
**Solution:** No, the potential function does not have a maximum or minimum in free space, it is because the absence of atmosphere around conductor prevents the phenomenon of electric discharge or potential leakage.

**Question 18.**

**A test charge q is made to move in the electric field of a point charge Q along two different closed paths [figure first path has sections along and perpendicular to lines of electric field]. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?**
**Solution:** Work done will be zero in both the cases.
Explanation: The electrostatic field is conservative, and in this field work done by electric force on the charge in a closed loop is zero. In this question both are closed paths, hence the work done in both the cases will be zero.

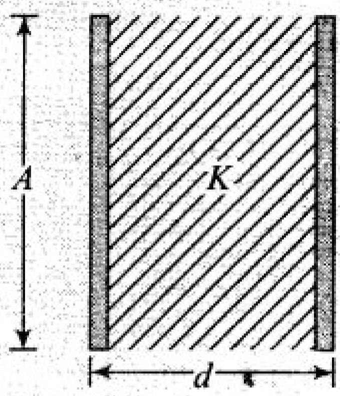
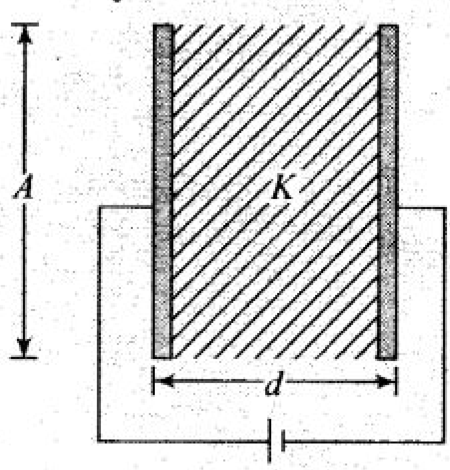
**Short Answer Type Questions**




**Question 19. Prove that a closed equipptential surface with no charge within itself must enclose an equipotential volume.**
**Solution:** Let us assume that in a closed equipotential surface with no charge the potential is changing from position to position. Let the potential just inside the surface is different to that of the surface causing in a potential gradient (dV/dr)
It means $E \neq 0$ electric field comes into existence, which is given by as $E=-dV/dr$
It means there will be field lines pointing inwards or outwards from the surface. These lines cannot be again on the surface, as the surface is equipotential. It is possible only when the other end of the field lines are originated from the charges inside. This contradicts the original assumption. Hence, the entire volume inside must be equipotential.

**Question 20. A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.**
**Solution:** The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is given by $C=K\epsilon_0 A/d$
The capacitance of the parallel plate capacitor decreases with the removal of dielectric medium as for air or vacuum K = 1 and for dielectric K > 1.
If we disconnect the battery from capacitor, then the charge stored will remain the same due to conservation of charge.
The energy stored in an isolated charge capacitor $U = q^2/2C$ as q is constant, energy stored $U \propto 1/C$ .As C decreases with the removal of dielectric medium, therefore energy stored increases.
The potential difference across the plates of the capacitor is given by $V = q/C$
Since q is constant and C decreases which in turn increases V and therefore E increases as $E = V/d$.

Important point:

| Quantity | Battery is Removed | Battery Remains connected |
|---|---|---|
| |  |  |
| Capacity | $C' = KC$ | $C' = KC$ |
| Charge | $Q' = Q$ (Charge is conserved) | $Q' = KQ$ |
| Potential | $V' = V/K$ | $V' = V$ (Since Battery maintains the potential difference) |
| Intensity | $E' = E/K$ | $E' = E$ |
| Energy | $U' = U/K$ | $U' = UK$ |

**Question 21. Prove that, if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must intermediate in potential between that of the charged body and that of infinity.**
**Solution:** The electric field E = dV/dr suggests that electric potential decreases along the direction of electric field.
Let us take any path from the eharged'conductor to the uncharged conductor along the direction of electric field. Therefore, the electric potential decrease along this path.
Now, another path from the uncharged conductor to infinity will again continually lower the potential further. This ensures that the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

**Question 22. Calculate potential energy of a point charge -q placed along the axis due to a charge +Q uniformly distributed along a ring of radius R. Sketch PE, as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if -q is displaced slightly from the centre of the ring (along the axis)?**
**Solution:** The potential energy (U) of a point charge q placed at-potential V,U=qV In our case a negative charged particle is placed at the axis of a ring having charge Q. Let the ring has radius a,the electric potential at an axial distance z from the centre of the ring is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{z^2 + a^2}}$$

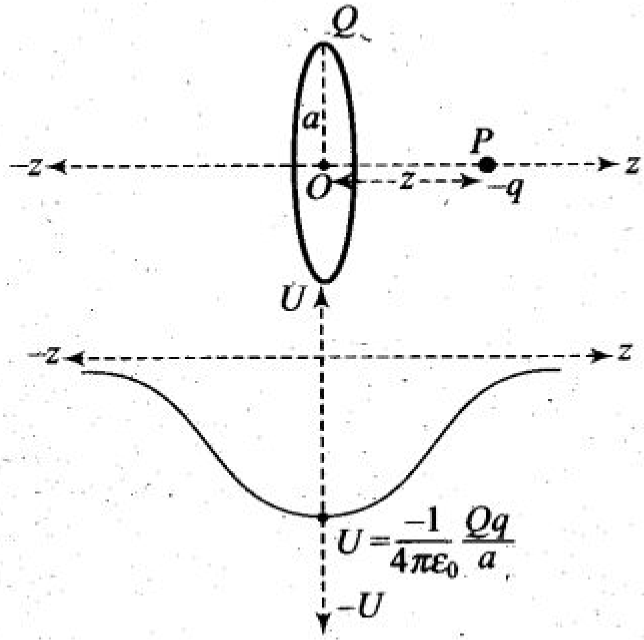
Hence potential energy of a point charge $-q$ is

$$U = qV = (-q)\left[\frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{z^2 + a^2}}\right]$$

$$\Rightarrow \quad U = -\frac{1}{4\pi\varepsilon_0} \frac{Qq}{\sqrt{z^2 + a^2}} = \frac{1}{4\pi\varepsilon_0 a} \frac{-Qq}{\sqrt{1 + \left(\frac{z}{a}\right)^2}}$$

At $z = 0$, $U = -\frac{1}{4\pi\varepsilon_0} \frac{Qq}{a}$;

At $z \to \infty$, $U \to 0$



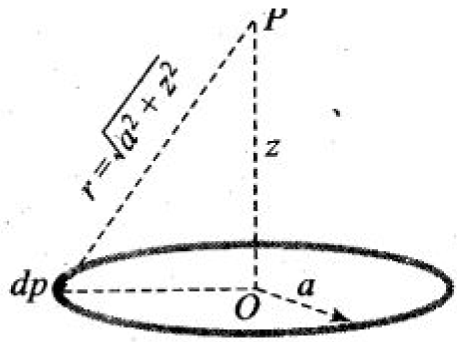

The variation of potential energy with z is shown in the figure.
The charge -q displaced would perform oscillations. Nothing can be concluded just by looking at the graph.

**Question 23. Calculate the potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.**

**Solution:**

Let us take point $P$ to be at a distance $z$ from the centre of the ring, as shown in figure.
The charge element $dq$ is at a distance $z$ from the point $P$. Therefore, $V$ can be written as

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+a^2}}$$




Since each element $dq$ is at the same distance from point $P$, so we have net potential

$$V = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+a^2}}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+a^2}}[Q]$$

The net electric potential $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{z^2+a^2}}$

**Long Answer Type Questions**

**Question 24. Find the equation of the equipotentials for an infinite cylinder of radius r0 carrying charge of linear density A.**
**Solution:** We know the integral relation between electric field gives potential difference between two points.
The electric field due to line charge need to be obtained in order to find the potential at distance r from the line chaige. For this we need to apply Gauss' theorem.
Let the field lines must be radically outward. Draw a cylindrical Gaussian surface of radius r and length l.

$$V(r) - V(r_0) = -\int_{r_0}^{r} \vec{\mathbf{E}} \cdot \mathbf{d\vec{r}} \qquad \ldots(i)$$

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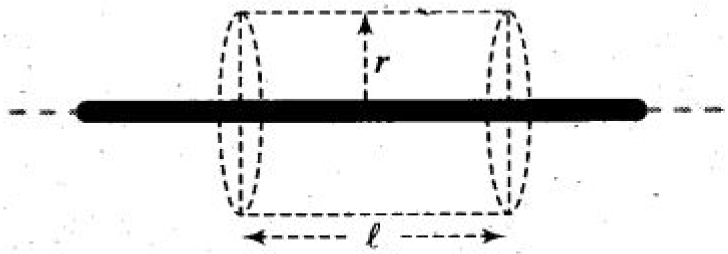



$$\oint \vec{\mathbf{E}} \cdot \mathbf{d\vec{S}} = \frac{1}{\varepsilon_0}(\lambda l)$$

or $$E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l \Rightarrow E_r = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \ldots(ii)$$

Hence, if $r_0$ is the radius of the cylindrical wire, then from (i) and (ii)

$$V(r) - V(r_0) = -\int_{r_0}^{r} \frac{\lambda}{2\pi\varepsilon_0 r} \cdot dr = -\frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r} \frac{dr}{r}$$

$$V(r) - V(r_0) = -\frac{\lambda}{2\pi\varepsilon_0}[\ln]_{r_0}^{r}$$

$$\Rightarrow \quad V(r) - V(r_0) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_0}$$

For a given $V$,

$$\ln\frac{r}{r_0} = -\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

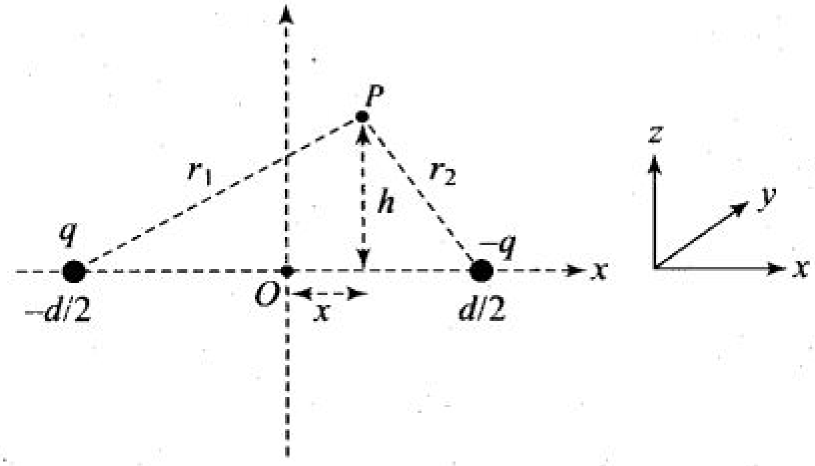
$$\Rightarrow \quad r = r_0 e^{-\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]}$$

The equipotential surfaces are cylinders of radius $r = r_0 e^{-\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]}$

>

**Question 25. Two point charges of magnitude +q and -q are placed at (-d/2, 0, 0) and (d/2, 2, 0), respectively. Find the equation of the equipotential surface where the potential is zero.**

**Solution:** Let the required plane lies at a distance x from the origin as shown in figure.




The potential at the point $P$ due to charges is given by

$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{r_2}$$

If net electric potential at this point is zero, then

$$0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2} \Rightarrow \frac{1}{r_1} = \frac{1}{r_2} \text{ or } r_1 = r_2$$

$$r_1 = \sqrt{(x + d/2)^2 + h^2} \text{ and } r_2 = \sqrt{(x - d/2)^2 + h^2}$$

or $\quad (x - d/2)^2 + h^2 = (x + d/2)^2 + h^2$

$\Rightarrow \quad x^2 - dx + d^2/4 = x^2 + dx + d^2/4$

or $\quad 2dx = 0 \Rightarrow x = 0$

The equation of the required plane is $x = 0$, i.e., $y$–$z$ plane.

**Question 26. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as ε= αU where α = 2V$^{-1}$. A similar capacitor with no dielectric is charged to $U_0$ = 78 V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.**

**Solution:** Both capacitors will be connected in parallel, hence the potential difference across both capacitors should be same. Assuming the required final voltage. be U. If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor is given by $Q_1$ = CU.

As the capacitor with the dielectric has a capacitance εC. Hence, the charge on the capacitor is given by

$$. Q_2 = \varepsilon CU = (\alpha U)CU = \alpha CU$$

The initial charge on the capacitor is given by

$$Q_0 = CU_0$$

From the conservation of charges, $Q_0 = Q_1 + Q_2 \Rightarrow CU_0 = CU + \alpha CU^2$

$$\Rightarrow \quad \alpha U_2 + U - U_0 = 0$$

$$\therefore \quad U = \frac{-1 \pm \sqrt{1 + 4\alpha U_0}}{2\alpha}$$

On solving for $U_0. = 78\ V$ and $\alpha = 2V^{-1}$,

$$U = \frac{-1 \pm \sqrt{1 + 4 \times 2 \times 78}}{2 \times 2} = \frac{-1 \pm \sqrt{1 + 624}}{4}$$

$$= \frac{-1 \pm \sqrt{625}}{4} = \frac{-1 \pm 25}{4} = -\frac{26}{4}\text{V and } 6\text{ V}$$

Hence final voltage,. $U = 6$ V

**Question 27. A capacitor is made of two circular plates of radius R each, separated by a , distance d << R. The capacitor is connected to a constant voltage. A thin conducting disc of radius r << R and thickness t << r is placed at the centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.**

**Solution:** Initially the thin conducting'disc is placed at the centre of the bottom plate, the potential of the disc will be equal to potential of the disc. The disc will be lifted if weight is balanced by electrostatic force.

The electric field on the disc, when potential difference V is applied across it

is given by $E = \frac{V}{d}$

Let charge $q'$ is transferred to the disc during the process,

$$\therefore \quad q' = -\varepsilon_0 \frac{V}{d} \pi r^2$$

The force acting on the disc is

$$F_{\text{electric}} = -\frac{V}{d} \times q' = \varepsilon_0 \frac{V^2}{d^2} \pi r^2$$

If the disc is to be lifted, then $F_{\text{electric}} = mg$

$$\varepsilon_0 \frac{V^2}{d^2} \pi r^2 = mg \Rightarrow V = \sqrt{\frac{mgd}{\pi \varepsilon_0 r^2}}$$

This is the required expression.

**Question 28. (a) In a quark model of elementary particles, a neutron is made of one up quarks [charge (2/3)e] and two down quarks [charges (-l/3)e]. Assume that they have a triangle configuration with side length of the order of $10^{-15}$ m. Calculate electrostatic potential energy of neutron and compare it with its mass 939 MeV.**

**(b) Repeat above exercise for a proton which is made of two up and one down quark.**
**Solution:** This system is made up of three charges. The potential energy of the system is equal to the algebraic sum of PE of each pair. So,

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_d q_d}{r} - \frac{q_u q_d}{r} - \frac{q_u q_d}{r}\right\}$$

$$= \frac{9\times10^9}{10^{-15}}(1.6\times10^{-19})^2\left[\left(\frac{1}{3}\right)^3 - \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\right]$$

$$= 2.304\times10^{-13}\left\{\frac{1}{9} - \frac{4}{9}\right\} = -7.68\times10^{-14}\text{ J}$$

$$= 4.8\times10^5\text{ eV} = 0.48\text{ MeV} = 5.11\times10^{-4}(m_n c^2)$$

**Question 29. Two metal spheres, one of radius R and the other of radius 2R, both have same surface charge density σ. They are brought in contact and separated. What will be the new surface charge densities on them?**
**Solution:** The charges on metal spheres before contact, are

$$Q_1 = \sigma\cdot 4\pi R^2$$

and $Q_2 = \sigma\cdot 4\pi(2R)^2 = 4(\sigma\cdot 4\pi R^2) = 4Q_1$

Let the charges on the metal spheres, after coming in contact becomes $Q'_1$ and $Q'_2$.

Applying law of conservation of charges,

$$Q'_1 + Q'_2 = Q_1 + Q_2 = 5Q_1 = 5(\sigma\cdot 4\pi R^2) \quad \ldots\text{(i)}$$

When metal spheres come in contact, they acquire equal potentials. Therefore, we have

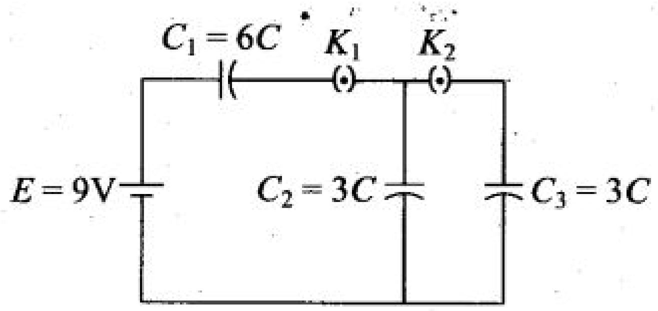
$$\frac{1}{4\pi\varepsilon_0}\frac{Q'_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q'_2}{2R} \Rightarrow Q'_1 = \frac{Q'_2}{2} \quad \ldots\text{(ii)}$$

On solving (i) and (ii), we get

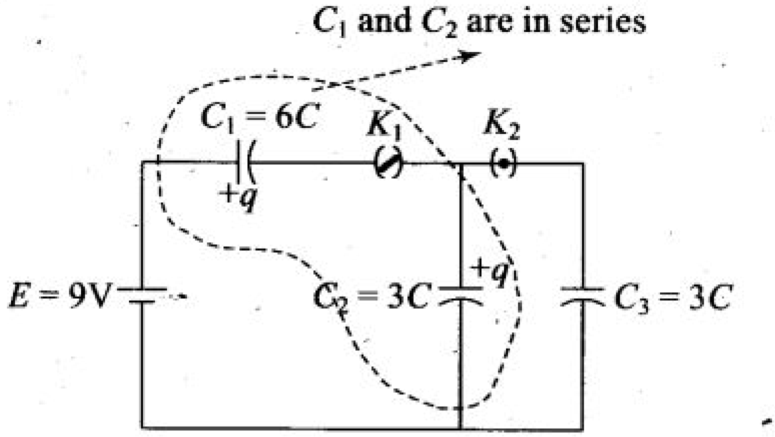
$$\therefore \quad Q'_1 = \frac{5}{3}(\sigma\cdot 4\pi R^2) \text{ and } Q'_2 = \frac{10}{3}(\sigma\cdot 4\pi R^2)$$

$$\therefore \quad \sigma_1 = \frac{5\sigma}{3} \quad \text{and} \quad \sigma_2 = \frac{5}{6}\sigma$$

**Question 30. In the circuit shown in figure, initially $K_1$ is closed and $K_2$ is open. What are the charges on each capacitors?**
**Then $K_1$ was opened and $K_2$ was closed (order is important), what will be the charge on each capacitor now? [C = 1 μF]**




**Solution:** In the circuit, when initially $K_1$ is closed and $K_2$ is open, the capacitors $C_1$ and $C_2$ connected in series with battery acquire equal charge.

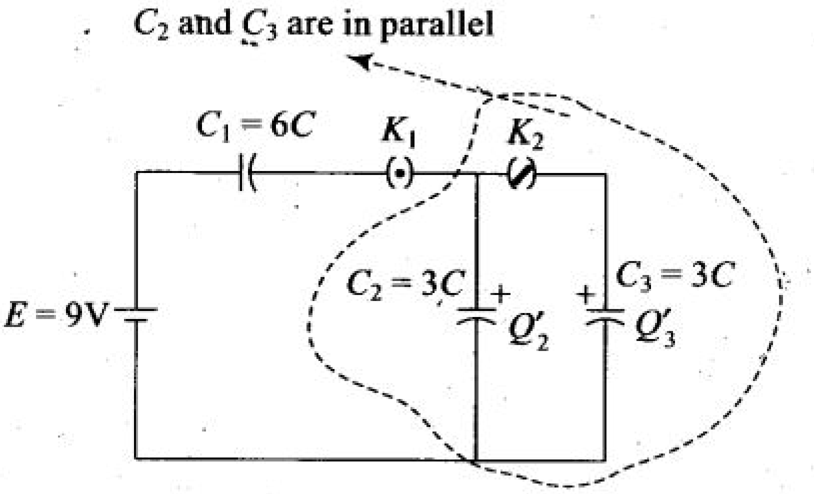



Hence the charge in capacitors $C_1$ and $C_2$ are

$$Q_1 = Q_2 = q = \left(\frac{C_1C_2}{C_1 + C_2}\right)E = \left(\frac{6C \times 3C}{6C + 3C}\right) \times 9 = 18\,\mu C$$

Hence the charge in capacitors $C_1$ and $C_2$ are
Henee $Q_1 = Q_2 = 18$ μC and $Q_3 = 0$
Now $K_1$ was opened and $K_2$ was closed, the battery and capacitor C, are disconnected from the circuit .The charge in capacitor C, will remain constant equal to $Q_1 – Q_2 = 18$ μC . The charged capacitor $C_2$ now connects in parallel with uncharged capacitor $C_3$, considering common

potential of parallel combination as V.




Then, $C_2V' + C_3V' = Q_2$

$$\Rightarrow \quad V' = \frac{Q_2}{C_2 + C_3} = \frac{18}{3C + 3C} = 3\text{V}$$

Hence $\quad Q_2' = 3CV' = 9\ \mu\text{C}$

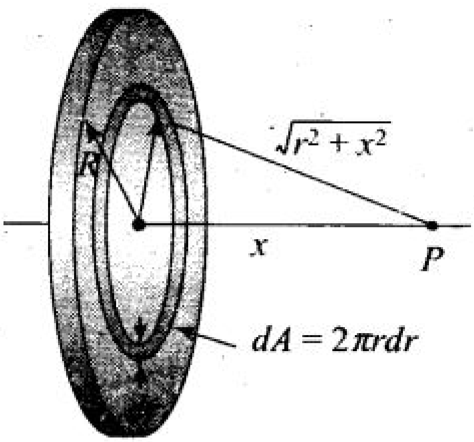
Also $\quad Q_3' = 3CV' = 9\ \mu\text{C}$

and $\quad Q_1' = 18\ \mu\text{C}$

**Question 31. Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.**

**Solution:**

Let us consider a point $P$ on the axis of the disc at a distance $x$ from the centre of the disk and take the plane of the disk to be perpendicular to the $x$-axis. Let the disc is divided into a number of charged rings as shown in figure.




The electric potential of each ring, of radius $r$ and width $dr$, have charge $dq$ is given by

$$dq = \sigma dA = \sigma 2\ \pi r dr$$

and potential is given by

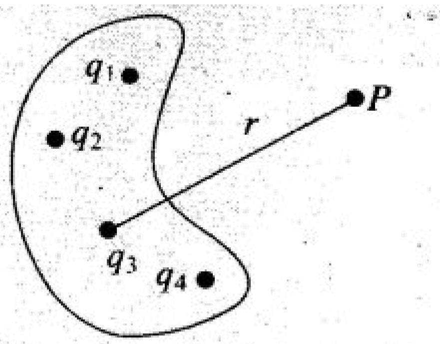
$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{\sqrt{r^2 + x^2}} = \frac{1}{4\pi\varepsilon_0} \frac{(\sigma 2\pi r\, dr)}{\sqrt{r^2 + x^2}}$$

The total electric potential at $P$, is given by

$$V = \frac{\sigma}{4\varepsilon_0} \int_0^R \frac{2r\, dr}{\sqrt{r^2 + x^2}} = \frac{\sigma}{4\varepsilon_0} \left[ \frac{\sqrt{r^2 + x^2}}{1/2} \right]_0^R$$

$$\Rightarrow \quad V = \frac{\sigma}{2\varepsilon_0} [\sqrt{R^2 + x^2} - x] = \frac{Q}{2\pi\varepsilon_0 R^2} [\sqrt{R^2 + x^2} - x]$$

**Question 32. Two charges $q_1$ and $q_2$ are placed at (0, 0, d) and (0, 0, -d) respectively. Find the locus of points where the potential is zero.**
**Solution:**
Key concept: Following the principle of superposition of potentials as described in last section, let us find the potential V due to a collection of discrete point charges $q_1$, $q_2$, …,$q_n$, at a point P.




The potential at P due to the system of point charges is given as the sum of their individual potentials at $P$, $V = \frac{1}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$

As we know, the potential at point $P$ is $V = \Sigma V_i$,

where $V_i = \frac{q_i}{4\pi\varepsilon_0}$ ; $r_i$ = magnitude of position vector $P$ relative to $q_i$.

Then $\quad V = \frac{1}{4\pi\varepsilon_0}\Sigma\frac{q_i}{r_{pi}}$

Let us take a point on the required plane as $(x, y, z)$. The two charges lies on $z$-axis at a separation of $2d$. The potential at the point $P$ due to two charges is given by

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$

$$\therefore \quad \frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} = \frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}}$$

On squaring and simplifying, we get

$$x^2+y^2+z^2+\left[\frac{(q_1/q_2)^2+1}{(q_1/q_2)^2-1}\right](2zd)+d^2-0$$

The standard equation of sphere is

$$x^2+y^2+z^2+2ux+2uy+2wz+g=0$$

with centre $(-u, -v, -w)$ and radius

$$\sqrt{u^2+v^2+w^2-g}$$

Hence centre of sphere will be

$$\left(0, 0, -d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]\right)$$

And radius is

$$r = \sqrt{\left(d\left[\frac{q_1^2+q_2^2}{q_1^2-q_2^2}\right]\right)^2 - d^2} = \frac{2q_1q_2d}{q_1^2-q_2^2}$$

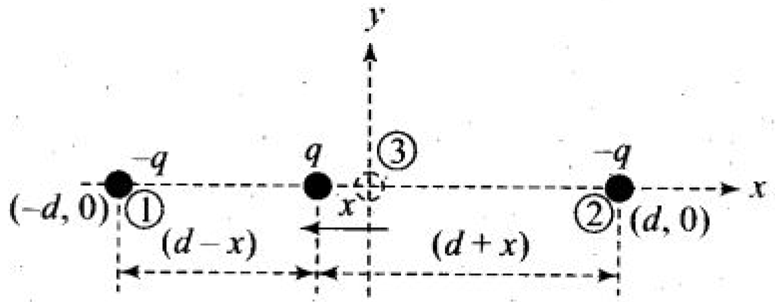
**Question 33.**

Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid-point $O$. Find the potential energy of $+q$ as a function of small distance $x$ from $O$ due to $-q$ charges. Sketch P.E v/s $x$ and convince yourself that the charge at $O$ is in an unstable equilibrium.

**Solution:**

Let third charge $+q$ is slightly displaced from $O$ towards the charge placed at $(-d, 0)$. We can write the potential energy of charge 3 as

$$U = q(V_1 + V_2) = q\left\{\frac{1}{4\pi\varepsilon_0}\frac{-q}{(d-x)} + \frac{-q}{(d+x)}\right\}$$



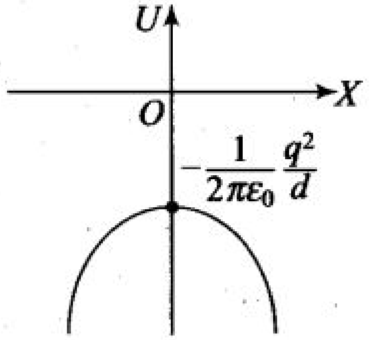

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q^2}{(d-x)} + \frac{-q^2}{(d+x)}\right\}$$

$$U = \frac{-q^2}{4\pi\varepsilon_0}\frac{2d}{(d^2 - x^2)} = \frac{1}{2\pi\varepsilon_0}\frac{-q^2 d}{(d^2 - x^2)} \quad \ldots\text{(i)}$$

At $x=0,\ U=-\frac{1}{2\pi\varepsilon_0}\frac{q^2}{d}$

Differentiating (i) w.r.t $x$

$$\frac{dU}{dx}=\frac{-q^2 2d}{4\pi\varepsilon_0}\cdot\frac{2x}{(d^2-x^2)^2} \quad \text{...(ii)}$$




At $x<0$, $\frac{dU}{dx}>0$

And at $x>0$, $\frac{dU}{dx}<0$

We define force on charge particle, $F=-\frac{dU}{dx}$

The system will be in equilibrium, if

$$F=-\frac{dU}{dx}=0$$

On solving, $x=0$ is the equilibrium position. For charge $+q$ to be in stable/unstable equilibrium,if

$\frac{d^2U}{dx^2}$ = positive, equilibrium is stable

$\frac{d^2U}{dx^2}$ = negative, equilibrium is unstable

$\frac{d^2U}{dx^2}$ = 0, equilibrium is neutral

Now differentiating eq. (ii) again

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left[\frac{2}{(d^2-x^2)^2} - \frac{8x^2}{(d^2-x^2)^3}\right]$$

$$= \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\frac{1}{(d^2-x^2)^3}[2(d^2-x^2)^2 - 8x^2]$$

At $x = 0$, $\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{d^6}\right)(2d^2) < 0$

This shows that system will be unstable equilibrium.

**Important points:**

- *Attractive force* :On increasing $x$, if $U$ increases $\frac{dU}{dx}$ = positive then $F$ is negative in direction, i.e. force is attractive in nature.
- *Repulsive force* : On increasing $x$, if $U$ decreases $\frac{dU}{dx}$ = negative then $F$ is positive in direction,i.e. force is repulsive in nature.
- *Zero force* : On increasing $x$, if $U$ does not change $\frac{dU}{dx} = 0$