# Chapter 11 - Dual Nature of Radiation and Matter

**Multiple Choice Questions (MCQs)**

**Single Correct Answer Type**
**Question 1. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to (a) H (b) $H^{1/2}$ (c) $H^0$ (d) $H^{-1/2}$**
**Solution:** (d)
Key concept: According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle.
The wave associated with a moving particle is called matter wave or de-Broglie wave and it propagates in the form of wave packets with group velocity. According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where $h$ = Planck's constant, $m$ = Mass of the particle, $v$ = Speed of the particle, $E$ = Energy of the particle.

For a body falling freely under gravity from a height $H$, final velocity of $v$ is obtained by applying kinematic equations

$$v^2 - u^2 = 2aH \Rightarrow v^2 - 0^2 = 2gH.$$

or, $v = \sqrt{2gH}$

We define de-broglie wavelength as

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}}$$

$$\Rightarrow \quad \lambda = \frac{h}{m\sqrt{2g}\sqrt{H}} = K\frac{1}{\sqrt{H}}$$

Where $K = \frac{h}{m\sqrt{2g}}$ = Constant

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{H}}.\ \text{Hence}\ \lambda \propto H^{1/2}$$

**Question 2. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly**
**a) 1.2 nm (b) 1.2 x $10^{-3}$ nm**
**(c) 1.2 x $10^{-6}$ nm (d). 1.2 x 10 nm**
**Solution:** (b)
Key concept: According to Einstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy.

Energy of photon is given by

$E = h\nu = \frac{hc}{\lambda}$; where $c$ = Speed of light, $h$ = Planck's constant = $6.6 \times 10^{-34}$ J-sec, $\nu$ = Frequency in Hz, $\lambda$ = the minimum wavelength of the photon required to eject the proton from nucleus.

In electron volt, $E(eV) = \frac{hc}{e\lambda} = \frac{12375}{\lambda(\text{Å})} \approx \frac{12400}{\lambda(\text{Å})}$

According to the problem,

Energy of a photon, $E$ = 1 MeV or $10^6$ eV

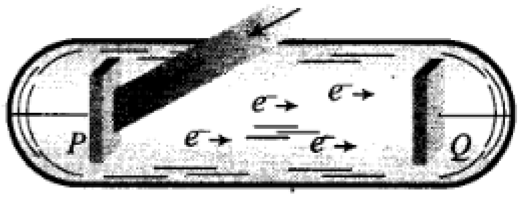
Now, $hc = 1240$ eV nm

Now, $E = \frac{hc}{\lambda}$

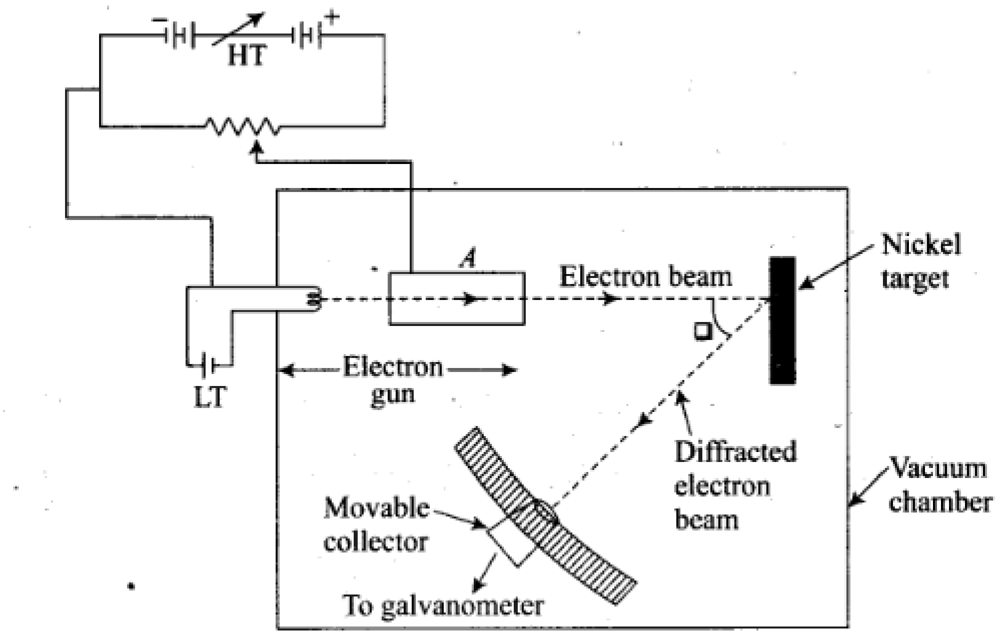
$$\Rightarrow \lambda = \frac{hc}{E} = \frac{1240}{10^6} \text{ nm}$$
$$= 1.24 \times 10^{-3} \text{ nm}$$

**Question 3. Consider a beam of electrons (each electron with energy $E_0$) incident on a metal surface kept in an evacuated chamber. Then,**
**(a) no electrons will be emitted as only photons can emit electrons**
**(b) electrons can be emitted but all with an energy, $E_0$**
**(c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ ($\phi$ is the work function)**
**(d) electron can be omitted with energy ,with a maximum of $E_0$**
**Solution:** (d) If a beam of electrons of having energy $E_0$ is incident on a metal surface kept in an evacuated chamber.

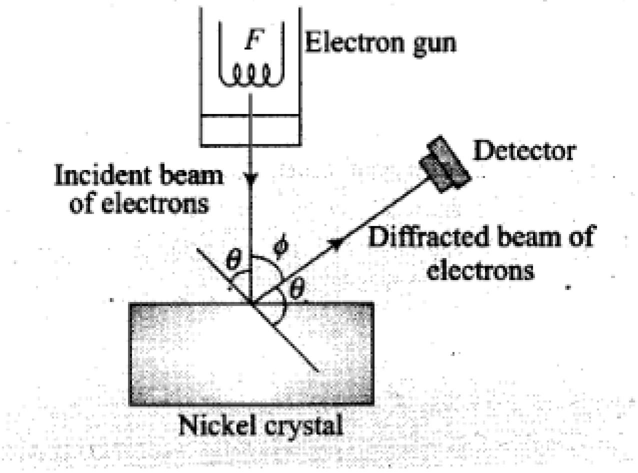
The electrons can be emitted with maximum energy $E_0$ (due to elastic collision) and With any energy less than $E_0$, when part of incident energy of electron is used in liberating the electrons from the surface of metal.




**Question 4. Consider the figure given below.**
**Suppose the voltage applied to A is increased. The diffracted beam will have the maximum at a value of θ that (a) will be larger than the earlier value**
**(b) will be the same as the earlier value**
**(c) will be less than the earlier value**
**(d) will depend on the target**

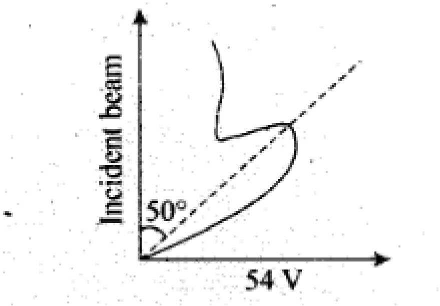



**Solution:** (c)
Key concept: Davision and Germer Experiment:
1. It is used to study the scattering of electron from a solid or to verify the wave nature of electron. A beam of electrons emitted by an . electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. Ni crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.




2. The diffracted beam of electrons is received by the detector which can be positioned at any angle by rotating it .about the point of incidence. The energy of the incideni beam of electrons can also be varied by changing the applied voltage to the electron gun.
According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering. It is maximum for diffracting angle 50° at 54 volt potential difference.
3. If the de-Broglie waves exist for electrons then these should be diffracted as X-rays. Using the Bragg's formula 2d sinθ = n**λ**, we can determine the wavelength of these waves.
The de-Broglie wavelength associated with electron is where V is the applied voltage.
Using the Bragg's formula we can determine the wavelength of these waves. If there is a maxima of the, diffracted electrons at an angle θ, then
2d sin θ = A (ii)
From Eq. (i), we note that if V is inversely proportional to the wavelength **λ**. i.e., V will increase with the decrease, in **λ**.
From Eq. (ii), we note that wavelength **λ** is directly proportional to sinθ and hence θ.
So, with the decrease in **λ** , θ will also decrease.

Thus, when the voltage applied to A is increased. The diffracted beam will have the maximum at a value of θ that will be less than the earlier value.




4. A proton, a neutron, an electron and an a-particle have same energy. Then, their de-Broglie wavelengths compare as

The de-Broglie wavelength associated with electron is

$$\lambda = \frac{12.27}{\sqrt{V}} \text{Å} \qquad \ldots(i)$$

where $V$ is the applied voltage.
Using the Bragg's formula we can determine the wavelength of these waves.
If there is a maxima of the diffracted electrons at an angle $\theta$, then

$$2d \sin \theta = \lambda \qquad \ldots(ii)$$

From Eq. (i), we note that if $V$ is inversely proportional to the wavelength $\lambda$.
i.e., $V$ will increase with the decrease in $\lambda$.
From Eq. (ii), we note that wavelength $\lambda$ is directly proportional to $\sin \theta$ and hence $\theta$.
So, with the decrease in $\lambda$, $\theta$ will also decrease.
Thus, when the voltage applied to $A$ is increased. The diffracted beam will have the maximum at a value of $\theta$ that will be less than the earlier value.

**Question 5. A proton, a neutron, an electron and an a-particle have same energy. Then, their de-Broglie wavelengths compare as (a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$**
**(c) $\lambda_e < \lambda_p = \lambda n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda n = \lambda_\alpha$**
**Solution:** (b)
Key concept:
• Matter Waves (de-Broglie Waves)
According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a

particle.

- **de-Broglie wavelength:** According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where $h$ = Plank's constant, $m$ = Mass of the particle, $v$ = Speed of the particle, $E$ = Energy of the particle.

The smallest wavelength whose measurement is possible is that of $\gamma$-rays.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, $\alpha$-particle etc. is of the order of $10^{-10}$ m.

We conclude from above that the relation between de-Broglie wavelength $\lambda$ and kinetic energy $K$ of the particle is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Here, for the given value of energy $K$, $\frac{h}{\sqrt{2K}}$ is a constant.

Thus, $\lambda \propto \frac{1}{\sqrt{m}}$

$\therefore \quad \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha = \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$

Comparing wavelength of proton and neutron, $m_p = m_n$, hence $\lambda_p = \lambda_n$.

As, $m_\alpha > m_p$ therefore $\lambda_\alpha < \lambda_p$

As, $m_e > m_n$ therefore $\lambda_e < \lambda_n$.

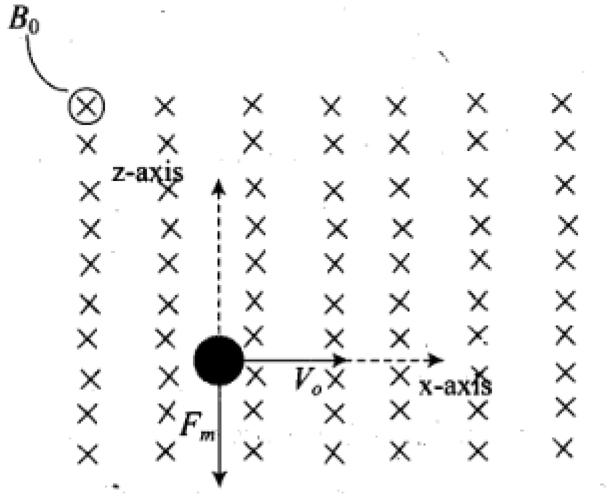
Hence, $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$.

**Question 6. An electron is moving with an initial velocity v = $v_0$i and is in a magnetic field B = $B_0$j. Then, its de-Broglie wavelength**
**(a) remains constant**
**(b) increases with time**
**(c) decreases with time**
**(d) increases and decreases periodically**
**Solution:** (a)
Key concept: If a particle is carrying a positive charge q and moving with a velocity v enters a magnetic field .5 then it experiences a force F which is given by the expression
F = q(v x B)=$ F = qvB sin θ. As this force is perpendicular to v and B , so the magnitude of v will

not change, i.e. momentum (p = mv) will remain constant in magnitude. Hence,

According to the problem, $\vec{v} = v_0 i$ and $\vec{B} = B_0 j$

Magnetic force on moving electron $= -e[v_0 i \times B_0 j] \Rightarrow = -ev_0 B_0 k$




As this force is perpendicular to $\vec{v}$ and $\vec{B}$, so the magnitude of $v$ will not change, i.e. momentum ($p = mv$) will remain constant in magnitude. Hence,

de-Broglie wavelength $\lambda = \dfrac{h}{mv}$ remains constant.

**Question 7. An electron (mass m) with an initial velocity $v = v_0 i (v_0 > 0)$ is in an electric field $E = E_0 \hat{i}$ ($E_0$ = constant > 0). Its de-Broglie wavelength at time t is given by**

(a) $\dfrac{\lambda_0}{\left[1+\dfrac{eE_0}{m}\dfrac{t}{v_0}\right]}$

(b) $\lambda_0\left[1+\dfrac{eE_0 t}{mv_0}\right]$

(c) $\lambda_0$

(d) $\lambda_0 t$

**Solution:** (a)

Key concept: The wave associated with moving particle is called matter wave or de-Broglie wave and it propagates in the form of wave packets with group velocity. According to de-Broglie theory, the wavelength of de- Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

*As initial velocity of the electron is* $v_0\hat{i}$ , the initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0} \qquad \text{...(i)}$$

Electrostatic force on electron in electric field is,

$$\vec{F}_e = -e\vec{E} = -e[-E_0\hat{i}] = eE_0\hat{i}$$

Acceleration of electron, $\vec{a} = \frac{\vec{F}}{m} = \frac{eE_0\hat{j}}{m}$

Velocity of the electron after time $t$,

$$\vec{v} = v_0\hat{i} + \left(\frac{eE_0\hat{i}}{m}\right)t = \left(v_0 + \frac{eE_0}{m}t\right)\hat{i}$$

$$\Rightarrow \quad \vec{v} = v_0\left(1 + \frac{eE_0}{mv_0}t\right)\hat{i}$$

de-Broglie wavelength associated with electron at time $t$ is $\lambda = \frac{h}{mv}$

$$\Rightarrow \quad \lambda = \frac{h}{m\left[v_0\left(1 + \frac{eE_0}{mv_0}t\right)\right]} = \frac{h/mv_0}{\left(1 + \frac{eE_0}{mv_0}t\right)}$$

$$\Rightarrow \quad \lambda = \frac{\lambda_0}{\left[1 + \frac{eE_0}{mv_0}t\right]} \qquad \text{As } \lambda_0 = \frac{h}{mv_0}$$

**Question 8. An electron (mass m) with an initial velocity v =$v_0\hat{i}$ is in an electric field** $\boldsymbol{E} = \boldsymbol{E_0}\hat{j}$. If $\lambda_0 = \frac{h}{mv_0}$, its de-Broglie wavelength at time $t$ is given by

(a) $\lambda_0$

(b) $\lambda_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$

(c) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$

(d) $\frac{\lambda_0}{\left(1 + \frac{e^2E_0^2t^2}{m^2v_0^2}\right)}$

**Solution:** (c) According to the problem de-Broglie wavelength of electron at time t=0.

is $\lambda_0 = \frac{h}{mv_0}$.

Electrostatic force on electron in electric field is

$$\vec{F}_e = -e\vec{E} = -eE_0\hat{j}$$

The acceleration of electron, $\vec{a} = \frac{\vec{F}}{m} = -\frac{eE_0}{m}\hat{j}$

It is acting along negative $y$-axis.

The initial velocity of electron along $x$-axis, $v_{x_0} = v_0\hat{i}$.

This component of velocity will remain constant as there is no force on electron in this direction.

Now considering $y$-direction. Initial velocity of electron along $y$-axis, $v_{y_0} = 0$.

Velocity of electron after time $t$ along $y$-axis,

$$v_y = 0 + \left(-\frac{eE_0}{m}\hat{j}\right)t = -\frac{eE_0}{m}t\hat{j}$$

Magnitude of velocity of electron after time $t$ is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(\frac{-eE_0}{m}t\right)^2}$$

$$\Rightarrow \quad = v_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$$

de-Broglie wavelength, $\lambda' = \frac{h}{mv}$

$$= \frac{h}{mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}} = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}}$$

$$\Rightarrow \quad \lambda' = \frac{\lambda_0}{\left(1 + \frac{e^2E_0^2t^2}{m^2v_0^2}\right)}$$

**One or More Than One Correct Answer Type**

**Question 9. Relativistic corrections become necessary when the expression for the kinetic energy 1/2 $mv^2$ , becomes comparable with $mc^2$. where m is the mass of the particle. At what de-Broglie wavelength, will relativistic corrections become important for an electron?**
**(a) A=10nm (b) A =$10^{-1}$ nm (c) A=$10^{-4}$ nm (d) A=$10^{-6}$ nm**
**Solution:** (c, d)
Key concept: De-Brogile or matter wave is independent of die charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).
The de-Broglie wavelength at which relativistic corrections become important that the phase velocity of the matter waves can be greater than the speed of the light (3 x $10^8$ m/s).
The wavelength of de-Broglie wave is given by
$\lambda$ = h/p = h/mv
Here, h = 6.6 x $10^{-34}$ Js
and for electron, m = 9 x $\mathbf{10^{-31}}$ kg

To approach these types of problem we use hit and trial method by picking up each option one by one.

In option (a), $\lambda_1 = 10 \text{ nm} = 10 \times 10^{-9} \text{ m} \simeq 10^{-8} \text{ m}$

$$\Rightarrow \quad v_1 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-8}}$$

$$= \frac{2.2}{3} \times 10^5 \approx 10^5 \text{ m/s}$$

In option (b), $\lambda_2 = 10^{-1} \text{ nm} = 10^{-1} \times 10^{-9} \text{ m} = 10^{-10} \text{ m}$

$$\Rightarrow \quad v_2 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-10}} \approx 10^7 \text{ m/s}$$

In option (c), $\lambda_3 = 10^{-4} \text{ nm} = 10^{-4} \times 10^{-9} \text{ m} = 10^{-13} \text{ m}$

$$\Rightarrow \quad v_3 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-13}} \approx 10^{10} \text{ m/s}$$

In option (d), $\lambda_4 = 10^{-6} \text{ nm} = 10^{-6} \times 10^{-9} \text{ m} = 10^{-15} \text{ m}$

$$\Rightarrow \quad v_4 = \frac{6.6 \times 10^{-34}}{(9 \times 10^{-31}) \times 10^{-15}} \approx 10^{12} \text{ m/s}$$

Thus, options (c) and (d) are correct as $v_3$ and $v_4$ is greater than $3 \times 10^8$ m/s.

**Question 10. Two particles $A_1$ and $A_2$ of masses $m_1$ , $m_2$ ( $m_1 > m_2$) have the same de-Broglie wavelength. Then,**
**(a) their momenta are the same (b) their energies are the same**
**(c) energy of $A_1$ is less than the energy of $A_2$**
**(d) energy of $A_1$ is more than the energy of $A_2$**
**Solution:** (a. c)

We know that de-Broglie wavelength $\lambda = \frac{h}{mv}$

where, $mv = p$ (momentum) of the particle

$\Rightarrow$ But we can express wavelength $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$
Here, $h$ is Planck constant.

Hence, $p \propto \frac{1}{\lambda} \Rightarrow \frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1}$

But particles have the same de-Broglie wavelength. $(\lambda_1 = \lambda_2) = \lambda$.

Then, $\frac{p_1}{p_2} = \frac{\lambda}{\lambda} = 1 \Rightarrow p_1 = p_2$

Thus, their momenta is same.

Also, $E = \frac{1}{2}mv^2 = \frac{1}{2}mv^2\frac{m}{m}$

$$= \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$$

As $p$ is constant, $E \propto \frac{1}{m}$

$\therefore \quad \frac{E_1}{E_2} = \frac{m_2}{m_1} < 1 \Rightarrow E_1 < E_2$

**Important points:**
Some important characteristics of Matter Waves:

1. Matter wave represents the probability of finding a particle in space.
2. Matter waves are not electromagnetic in nature.
3. de-Brogile or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).
4. Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles.
5. Electron microscope works on the basis of de-Broglie waves.
6. The phase velocity of the matter waves can be greater than the speed of the light.
7. Matter waves can propagate in vacuum, hence they are not mechanical waves.
8. The number of de-Broglie waves associated with $n^{th}$ orbital electron is $n$.
9. Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

**Question 11. De-Broglie wavelength associated with uncharged particles: For Neutron efe-Broglie wavelength is given as $v_e$=c/100.Then**

(a) $\frac{E_e}{E_p} = 10^{-4}$ (b) $\frac{E_e}{E_p} = 10^{-2}$ (c) $\frac{p_e}{m_e c} = 10^{-2}$ (d) $\frac{p_e}{m_e c} = 10^{-4}$

**Solution:**(b,c)

**Key concept:** *De-Broglie wavelength associated with the charged particles:* The energy of a charged particle accelerated through potential difference $V$ is $E = \frac{1}{2}mv^2 = qV$

Hence de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$\lambda_{\text{Electron}} = \frac{12.27}{\sqrt{V}}\text{Å},\ \lambda_{\text{Proton}} = \frac{0.286}{\sqrt{V}}\text{Å},$$

$$\lambda_{\text{Deutron}} = \frac{0.202}{\sqrt{V}}\text{Å},\ \lambda_{\alpha\text{-particle}} = \frac{0.101}{\sqrt{V}}\text{Å}$$

*De-Broglie wavelength associated with uncharged particles*: For *Neutron* de-Broglie wavelength is given as

$$\lambda_{\text{Neutron}} = \frac{0.286\times10^{-10}}{\sqrt{E(\text{in eV})}}m = \frac{0.286}{\sqrt{E(\text{in eV})}}\text{Å}$$

Energy of thermal neutrons at ordinary temperature

$\because E = kT \Rightarrow \lambda = \frac{h}{\sqrt{2mkT}}$ ; where $T$ = Absolute temperature,

$k$ = Boltzman's constant = $1.38\times10^{-23}$ Joule/kelvin

So, $$\lambda_{\text{Thermal neutron}} = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.38\times10^{-23}T}} = \frac{30.83}{\sqrt{T}}\text{Å}$$

Mass of electron = $m_e$, Mass of photon = $m_p$,
Velocity of electron is equal to velocity of proton which are $v_e$ and $v_p$ respectively.

de-Broglie wavelength for electron, $\lambda_e = \frac{h}{m_e v_e}$

$$\Rightarrow \quad \lambda_e = \frac{h}{m_e(c/100)} = \frac{100h}{m_e c} \qquad \text{...(i)}$$

Kinetic energy of electron, $E_e = \frac{1}{2}m_e v_e^2$

$$\Rightarrow \quad m_e v_e = \sqrt{2E_e m_e}$$

So, $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}}$

$$\Rightarrow \quad E_e = \frac{h^2}{2\lambda_e^2 m_e} \qquad \text{...(ii)}$$

The wavelength of proton is $\lambda_p$, and having an energy is

$$\therefore \quad E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e} \qquad [\because \lambda_p = 2\lambda_e]$$

$$\therefore \quad \frac{E_p}{E_e} = \left(\frac{hc}{2\lambda_e}\right)\left(\frac{2\lambda_e^2 m_e}{h^2}\right)$$

$$= \frac{\lambda_e m_e c}{h} = \frac{100h}{m_e c} \times \frac{m_e c}{h} = 100$$

So, $\frac{E_e}{E_p} = \frac{1}{100} = 10^{-2}$

For electron, $p_e = m_e v_e = m_e \times c/100$

So, $\frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2}$

**Important point:**

*Ratio of wavelength of photon and electron*: The wavelength of a photon of energy $E$ is given by $\lambda_{ph} = \frac{hc}{E}$

While the wavelength of an electron of kinetic energy $K$ is given by $\lambda_e = \frac{h}{\sqrt{2mK}}$.

Therefore, for the same energy, the ratio $\frac{\lambda_{ph}}{\lambda_e} = \frac{c}{E}\sqrt{2mK} = \sqrt{\frac{2mc^2K}{E^2}}$

**Question 12. Photons absorbed in a matter are converted to heat. A source emitting v photon/sec of frequency v is used to convert 1 kg of ice at 0°C to water at 0°C. Then, the time T taken for the conversion**

**(a) decreases with increasing n. with v fixed**

**(b) decreases with n fixed, v increasing**

**(c) remains constant with n and v changing such that nv = constant**

**(d) increases when the product nv increases**

**Solution:** (a, b. c).

We know that energy spent to convert ice into water

$E_{absorb}$ = mass × latent heat

$\therefore \quad E = mL = (1000\text{ g}) \times (80\text{ cal/g})$

$E = 80000\text{ cal}$

Energy of photons used $= nT \times E = nT \times h\nu$ $\quad [\because E = h\nu]$

So, $nTh\nu = mL \Rightarrow T = \dfrac{mL}{nh\nu}$

$\therefore \quad T \propto \dfrac{1}{n}$, when $\nu$ is constant.

Thus, time taken for conversion decreases with increasing $n$, with $\nu$ kept constant.

$T \propto \dfrac{1}{\nu}$, when $n$ remains constant.

Thus. time taken for conversion decreases with increasing $\nu$, with $n$ kept constant.

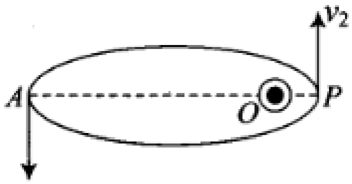
$\Rightarrow \quad T \propto \dfrac{1}{n\nu}$

Thus, time taken for conversion decreases with increase in product of $n\nu$ and $T$ is constant, if $n\nu$ is constant.

**Question 13. A particle moves in a closed orbit around the origin, due to a force which is directed towards the origin. The de-BrOglie wavelength of the particle varies cyclically between two values $\lambda_1$, $\lambda_2$ with $\lambda_1 > \lambda_2$ Which of the following statements are true?**
**(a) The particle could be moving in a circular orbit with origin as centre**
**(b) The particle could be moving in an elliptic orbit with origin as its focus**
**(c) When the de-Broglie wavetength is $\lambda_1$ the particle is nearer the origin than when its value is $\lambda_2$**
**(d) When the de-Broglie wavelength is $\lambda_2$ the particle is nearer the origin than when its value is $\lambda_1$**

**Solution:**

**(b, d)** According to the question, here given that the de-Broglie wavelength of the particle can be varying cyclically between two values $\lambda_1$ and $\lambda_2$, it is possible if particle is moving in an elliptical orbit with origin as its one focus.




As shown in the figure given alongside,

Let $v_1$ and $v_2$ be the speed of particle at $A$ and $B$ respectively and origin is at focus $O$. If $\lambda_1$ and $\lambda_2$ are the de-Broglie wavelengths associated with particle while moving at $A$ and $B$ respectively, then

$$\lambda_1 = \frac{h}{mv_1}$$

and $$\lambda_2 = \frac{h}{mv_2}$$

$\therefore$ $$\frac{\lambda_1}{\lambda_2} = \frac{v_2}{v_1}$$

Since $\lambda_1 > \lambda_2$

$\therefore$ $v_2 > v_1$

By law of conservation of angular momentum, the particle moves faster when it is closer to focus.

From figure, we note that origin $O$ is closed to $P$ than $A$.

**Very Short Answer Type Questions**

**Question 14. Aproton and an a-particle are accelerated, using the same potential difference. How are the de-Broglie wavelengths $\lambda_p$ and $\lambda_\alpha$ related to each other?**

**Solution:**

**Key concept:** Hence de-Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

In this problem since both proton and $\alpha$-particle are accelerated through same potential difference,

We know that, $\lambda = \dfrac{h}{\sqrt{2mqv}}$

$$\therefore \quad \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha q_\alpha}}{m_p q_p} = \frac{\sqrt{4m_p \times 2e}}{\sqrt{m_p \times e}} = \sqrt{8}$$

$$\therefore \quad \lambda_p = \sqrt{8}\lambda_\alpha$$

i.e., wavelength of proton is $\sqrt{8}$ times wavelength of α-particle.

**Important point:**

*De-Broglie wavelength associated with the charged particles* : The energy of a charged particle accelerated through potential difference $V$ is $E = \frac{1}{2}mv^2 = qV$

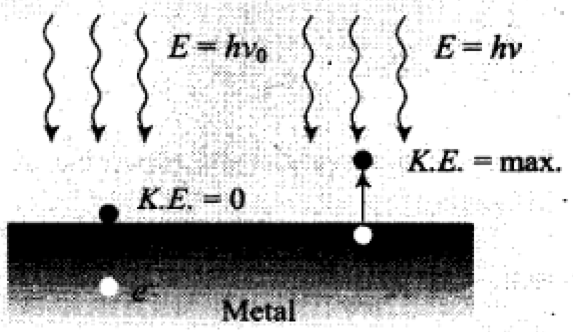
Hence de-Broglie wavelength $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2mqV}}$

$$\lambda_{\text{Electron}} = \frac{12.27}{\sqrt{V}}\text{Å}, \ \lambda_{\text{Proton}} = \frac{0.286}{\sqrt{V}}\text{Å}$$

$$\lambda_{\text{Deutron}} = \frac{0.202}{\sqrt{V}}\text{Å}, \ \lambda_{\alpha\text{-particle}} = \frac{0.101}{\sqrt{V}}\text{Å}$$

**Question 15. (i) In the explanation of photoeletric effect, we assume one photon of frequency v collides with an electron and transfers its energy. This leads . to the equation for the maximum energy $E_{max}$ of the emitted electron as $E_{max} = hv - \phi_0$ where $\phi_0$ is the work function of the metal. If an electron absorbs 2 photons (each of frequency v), what will be the maximum energy for the emitted electron?**
**(ii) Why is this fact (two photon absorption) not taken into consideration in our discussion of the stopping potential?**
**Solution:**

**Key concept:** *Einstein's photoelectric equation*: According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed.




Einstein's photoelectric equation is $E = W_0 + K_{max}$

where $K_{max} = \frac{1}{2}mv_{max}^2$ = maximum kinetic energy of emitted electrons.

And $W_0$ = Work function (or threshold energy)

$W_0 = h\nu_0 = \frac{hc}{\lambda_0}$ Joules; $\nu_0$ = Threshold frequency and $\lambda_0$ = threshold wavelength

(i) According to the question, an electron absorbs the energy of two photons each of frequency $\nu$ then $\nu' = 2\nu$ where $\nu'$ is the frequency of emitted electron.

Here, $E_{max} = h\nu - \phi_0$

Thus, maximum energy for emitted electrons is

$$E_{max} = h(2\nu) - \phi_0 = 2h\nu - \phi_0$$

(ii) The probability of absorbing two photons by the same electron is very low. Hence, such emission will be negligible.

**Question 16. There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of larger wavelength and emit light of shorter wavelength?**

**Solution:** In the first case, when the materials which absorb photons of shorter wavelength has the energy of the incident photon on the material is high and the energy of emitted photon is low when it has a longer wavelength or in short we can say that energy given out is less than the energy supplied.

But in second case, the energy of the incident photon is low for the substances which has to absorb photons of larger wavelength and energy of emitted photon is high to emit light Of shorter wavelength. This means in this statement material has to supply the energy for the emission of photons.

But this is not possible for a stable substances.

**Question 17. Do all the electrons that absorb a photon come out as photo electrons?**

**Solution:**

Key concept: Photo-Electric Effect:

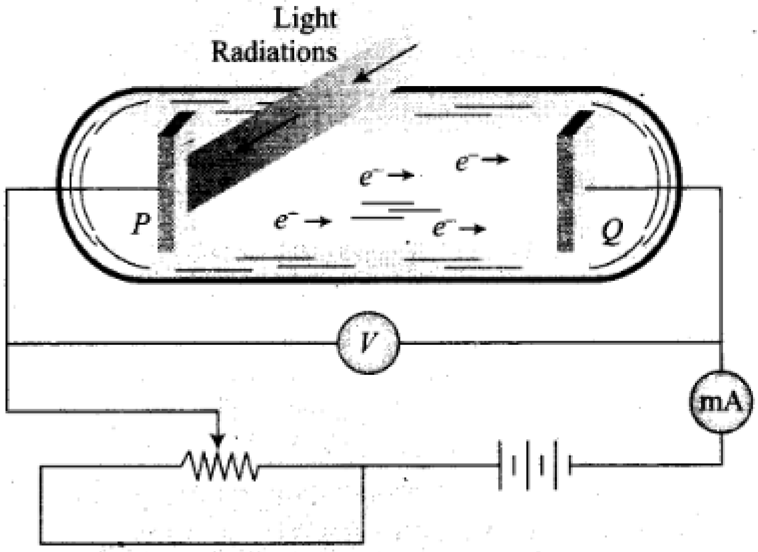
The photo-electtic effect is the emission of electrons (called photo-electrons when light strikes a surface. To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of positive ions in the material of the surface.

The photoelectric effect is based on the principle of conservation of energy.

1. Two conducting electrodes, the anode (Q) and cathode (P) are enclosed in an evacuated glass tube as shown on next page.

2. The battery or other source of potential difference creates an electric field in the direction from

anode to cathode.
3. Light of certain wavelength or frequency falling on the surface of cathode causes a current in the external circuit called photoelectric current.
4. As potential difference increases, photoelectric current also increases till saturation is reached.




5. When polarity of battery is reversed (i.e., plate $Q$ is at negative potential w.r.t. plate $P$) electrons start moving back towards the cathode.
6. At a particular negative potential of plate $Q$ no electron will reach the plate $Q$ and the current will become zero. This negative potential is called **stopping potential** denoted by $V_0$. Maximum kinetic energy of photo electrons in terms of stopping potential will therefore be $K_{max} = (|V_0|)$ eV

So we conclude that in photoelectric effect, we can observe that most electrons get scattered into the metal by absorbing a photon.

Therefore, all the electrons that absorb a photon doesn't come out as photoelectron. Only a few come out of metal whose energy becomes greater than the work function of metal.

**Question 18. There are two sources of light, each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength.**
**Solution:**
Key concept: X-Rays:
1. X-rays were discovered by scientist Roentgen that is why they are also called Roentgen rays.
2. Roentgen discovered that when pressure inside a discharge tube is kept 10“3 mm of Hg and potential difference is kept 25 kV, then some unknown radiations (X-rays) are emitted by anode.
3. There are three essential requirements for the production of X-rays.
(i) A source of electron
(ii) An arrangement to accelerate the electrons
(iii) A target of suitable material of high atomic weight and high melting point on which these high

speed electrons strike.

Here in this problem total energy will be constant.

Let us assume wavelength of X-rays is $\lambda_1$ and the wavelength of visible light is $\lambda_2$.

Given, $P = 100$ W

$\lambda_1 = 1$ nm

and $\lambda_2 = 500$ nm

Let $n_1$ and $n_2$ be the number of photons of X-rays and visible light emitted from the two sources per sec.

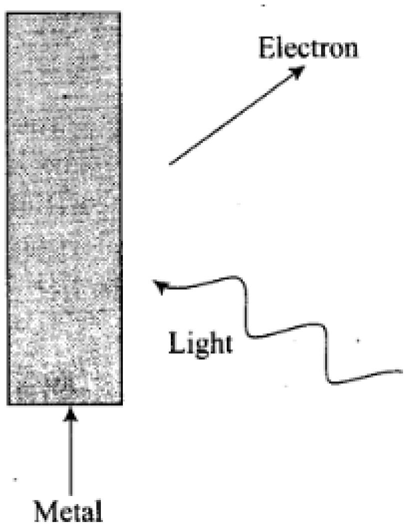
So, $$\frac{E}{t} = P = n_1 \frac{hc}{\lambda_1} = n_2 \frac{hc}{\lambda_2}$$

$$\Rightarrow \quad \frac{n_1}{\lambda_1} = \frac{n_2}{\lambda_2}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{500}$$

**Important point:** The wavelength of characteristic X-ray doesn't depend on accelerating voltage. It depends on the atomic number (Z) of the target material. In characteristic X-ray spectrum $\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$ and $\nu_{K\alpha} < \nu_{L\alpha} < \nu_{M\alpha}$. Also $\lambda_{K\alpha} < \lambda_{K\beta} < \lambda_{K\gamma}$

**Question 19. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.**




**Solution:**The momentum of incident photon is transferred to the metal ,during photo electric emission.

At microscopic level ,atoms of a metal absorb the photon and its momentum is transferred mainly to the nucleus and electrons.The excited electron is emitted.Therefore,the conservation of momentum is to be considered as the momentum of incident photon transferred to the nucleus and electron.

**Question 20.Consider a metal exposed to light the wavelength of 600 nm. The maximum energy of electron doubles when light of wavelength 400 nm is used.Find the work function**

**in eV?**
**Solution:**

**Key concept:** *Work function (or threshold energy) ($W_0$)*: The minimum energy of incident radiation required to eject the electrons from metallic surface is defined as work function of that surface.

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0} \text{ Joules; } \nu_0 = \text{Threshold frequency;}$$

$\lambda_0$ = Threshold wavelength

Work function in electron volt, $W_0(\text{eV}) = \dfrac{hc}{e\lambda_0} = \dfrac{12375}{\lambda_0(\text{Å})}$

Einstein's photoelectric equation is $E = W_0 + K_{max}$

Maximum energy = $h\nu - \phi$

According to the problem for first condition wavelength of light $\lambda = 600$ nm and for the second condition, wavelength of light $\lambda' = 400$ nm

Also, maximum kinetic energy for the second condition is equal to the twice of the kinetic energy in first condition.

i.e., $K'_{max} = 2K_{max}$

then $K'_{max} = \dfrac{hc}{\lambda} - \phi$

$\Rightarrow \quad 2K_{max} = \dfrac{hc}{\lambda'} - \phi_0$

$\Rightarrow \quad 2\left(\dfrac{1230}{600} - \phi\right) = \left(\dfrac{1230}{400} - \phi\right) \quad [\because hc \simeq 1240 \text{ eV nm}]$

$\Rightarrow \quad \phi = \dfrac{1230}{1230} = 1.02 \text{ eV}$

**Question 21. Assuming an electron is confined to a 1 nm wide region, find the uncertainty in momentum using Heisenberg uncertainty principle (Δx xΔp=h). You can assume the uncertainty in position Δx as 1 nm. Assuming p = Δp, find the energy of the electron in electron volts.**
**Solution:**

In this problem, $\Delta x = 1 \text{ nm} = 10^{-9}$ m, we have to find $\Delta p$. As we know $\Delta x \cdot \Delta p = h$

Therefore, $$\Delta p = \frac{h}{\Delta x} = \frac{h}{2\pi\Delta x}$$

$$\Rightarrow \quad = \frac{6.62 \times 10^{-34} \text{ Js}}{2 \times (22/7)(10^{-9}) \text{ m}}$$

$$= 1.05 \times 10^{-25} \text{ kg m/s}$$

Energy, $$E = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} \qquad [\because p \approx \Delta p]$$

$$= \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} \text{J}$$

$$\Rightarrow \quad = \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{eV}$$

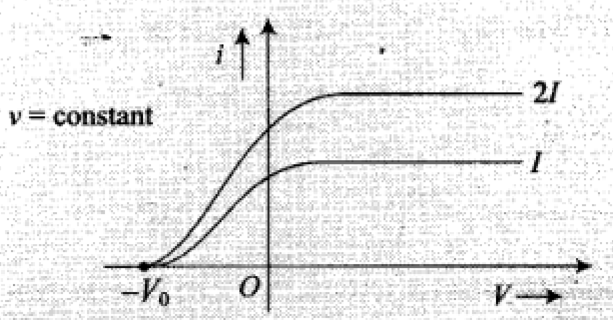
$$= 3.8 \times 10^{-2} \text{ eV}$$

**Question 22. Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies?**

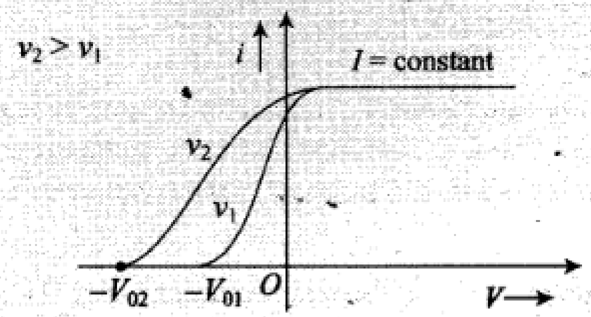
**Solution:**

**Key concept:** *Effect of intensity*: If the intensity of light is increased (while its frequency is kept the same) the current levels off at a higher value, showing that more electrons are being emitted per unit time. But the stopping potential $V_0$ doesn't change, i.e.

Intensity $\propto$ no. of incident photon $\propto$ no. of emitted photoelectron per time $\propto$ photo current.




*Effect of frequency*: If frequency of incident light increases, (keeping intensity constant) stopping potential increases but their is no change in photoelectric current.




Let us assume $n_A$ is the number of photons falling per second of beam $A$ and $n_B$ is the number of photons falling per second of beam $B$.

And it is given that the number of photons hitting the screen by beam $A$ is twice that by beam $B$. $n_A = 2n_B$

Energy of falling photon of beam $A = h\nu_A$

Energy of falling photon of beam $B = h\nu_B$

Now, according to question, intensity of $A$ is equal to intensity of $B$.

Therefore, $\quad I = n_A\nu_A = n_B\nu_B$

$$\Rightarrow \quad \frac{\nu_A}{\nu_B} = \frac{n_B}{n_A} = \frac{n_B}{2n_B} = \frac{1}{2}$$

$$\Rightarrow \quad \nu_B = 2\nu_A$$

**Question 23. Two particles A and B of de-Broglie wavelengths A, and combine to form a particle C. The process conserves momentum. Find the de-Broglie wavelength of the particle C. (The motion is one-dimensional)**

**Solution:**

By the law of conservation of momentum,

$$|p_C| = |p_A| + |p_B|$$

Let us first take the case I when both $p_A$ and $p_B$ are positive,

then $$\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

In second case when both $p_A$ and $p_B$ are negative,

then $$\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

In case III when $p_A > 0, p_B < 0$ *i.e.*, $p_A$ is positive and $p_B$ is negative,

$$\frac{h}{\lambda_C} = \frac{h}{\lambda_A} - \frac{h}{\lambda_B} = \frac{(\lambda_B - \lambda_A)h}{\lambda_A \lambda_B}$$

$$\Rightarrow \qquad \lambda_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A}$$

And in case IV when $p_A < 0, p_B > 0$, i.e., $p_A$ is negative and $p_B$ is positive.

$$\therefore \qquad \frac{h}{\lambda_C} = \frac{-h}{\lambda_A} + \frac{h}{\lambda_B}$$

$$\Rightarrow \qquad = \frac{(\lambda_A - \lambda_B)h}{\lambda_A \lambda_B} \Rightarrow \lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}$$

**Question 24. A neutron beam of energy E scatters from atoms on a surface with a spacing d = 0.1 nm. The first maximum intensity in the reflected beam occurs at θ= 30°. What is the kinetic energy E of the beam in eV?**

**Solution:**

By the law of conservation of momentum,

$$|p_C| = |p_A| + |p_B|$$

Let us first take the case I when both $p_A$ and $p_B$ are positive,

then $$\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

In second case when both $p_A$ and $p_B$ are negative,

then $$\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

In case III when $p_A \geq 0, p_B < 0$ *i.e.*, $p_A$ is positive and $p_B$ is negative,

$$\frac{h}{\lambda_C} = \frac{h}{\lambda_A} - \frac{h}{\lambda_B} = \frac{(\lambda_B - \lambda_A)h}{\lambda_A \lambda_B}$$

$$\Rightarrow \quad \lambda_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A}$$

And in case IV when $p_A < 0, p_B > 0$, i.e., $p_A$ is negative and $p_B$ is positive.

$$\therefore \quad \frac{h}{\lambda_C} = \frac{-h}{\lambda_A} + \frac{h}{\lambda_B}$$

$$\Rightarrow \quad = \frac{(\lambda_A - \lambda_B)h}{\lambda_A \lambda_B} \Rightarrow \lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}$$

Now, $$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{p^2}{m}$$

$$= \frac{1}{2} \times \frac{(6.62 \times 10^{-24})^2}{1.67 \times 10^{-27}} \text{J}$$

$$= 0.21 \text{ eV}$$

**Long Answer Type Questions**

**Question 25. Consider a thin target ($10^{-2}$ cm square, $10^{-3}$ m thickness) of sodium, which produces a photocurrent of 100 $\mu$A when a light of intensity 100 W/$m^2$ ($\lambda$ = 660 nm) falls on it. Find the probability that a photoelectron is produced when a photon strikes a sodium atom. [Take density of Na = 0.97 kg/$m^3$]**

**Solution:**

According to the problem, area of the target $A = 10^{-2}\ cm^2 = 10^{-4}\ m^2$

And thickness, $d = 10^{-3}$ m

Photo current, $i = 100 \times 10^{-6}\ A = 10^{-4}\ A$

Intensity, $I = 100\ W/m^2$

$\Rightarrow \quad \lambda = 660\ nm = 660 \times 10^{-9}$ m

$\rho_{Na} = 0.97\ kg/m^3$

Avogadro's number $= 6 \times 10^{26}$ kg atom

Volume of sodium target $= A \times d$

$= 10^{-4} \times 10^{-3}$

$\Rightarrow \quad = 10^{-7}\ m^3$

We know that $6 \times 10^{26}$ atoms of sodium weighs = 23 kg

Density of sodium = $0.97\ kg/m^3$

Hence the, volume of $6 \times 10^{6}$ sodium atoms $= \dfrac{23}{0.97}\ m^3$

$$\text{Volume occupied by one sodium atom} = \frac{23}{0.97 \times (6 \times 10^{26})} = 3.95 \times 10^{-26}\ m^3$$

$$\text{Number of sodium atoms in target } (N_{sodium}) = \frac{10^{-7}}{3.95 \times 10^{-26}} = 2.53 \times 10^{18}$$

Let $n$ be the number of photons falling per second on the target.

Energy of each photon $= hc/\lambda$

$$\text{Total energy falling per second on target} = \frac{nhc}{\lambda} = IA$$

$$\therefore \quad n = \frac{IA\lambda}{hc}$$

$$= \frac{100 \times 10^{-4} \times (660 \times 10^{-9})}{(6.62 \times 10^{-34}) \times (3 \times 10^{8})} = 3.3 \times 10^{16}$$

Let $P$ be the probability of emission per atom per photon.

The number of photoelectrons emitted per second

$$N = P \times n \times (N_{sodium})$$

$$= P \times (33 \times 10^{16}) \times (2.53 \times 10^{18})$$

Now, according to question,

$$i = 100\ \mu A = 100 \times 10^{-6} = 10^{-4}\ A$$

Current, $i = Ne$

$$\therefore \quad 10^{-4} = P \times (3.3 \times 10^{16}) \times (2.53 \times 10^{18}) \times (1.6 \times 10^{-19})$$

$$\Rightarrow \quad P = \frac{10^{-4}}{(3.3 \times 10^{16}) \times (2.53 \times 10^{18}) \times (1.6 \times 10^{-19})}$$

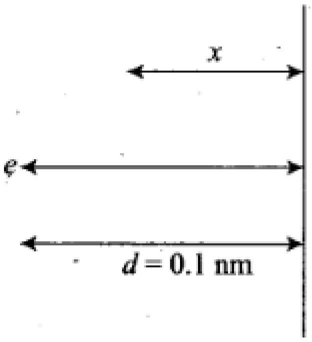
$$= 7.48 \times 10^{-21}$$

Then, the probability of photoemission by a single photon on a single atom is very much less than 1. Because, the absorption of two photons by an atom is negligible.

**Question 26. Consider an electron in front of metallic surface of a distance d.Assume the force of attraction by the plate is given as . Calculate work in taking the to an infinite distance from the plate .Taking d=0.1 nm. find the work done in electron volts?**

**Solution:**

Consider the figure in which an electron is displaced slowly by a distance $x$ by the means of an external force which is,




$$F = \frac{q^2}{4 \times 4\pi\varepsilon_0 d^2}$$

where, $d = 0.1\ \text{nm} = 10^{-10}\ \text{m}$

Let the electron be at distance $x$ from metallic surface. Then, force of attraction on it is

$$F_x = \frac{q^2}{4 \times 4\pi\varepsilon_0 d^2}$$

Work done by external agency in taking the electron from distance $d$ to infinity is

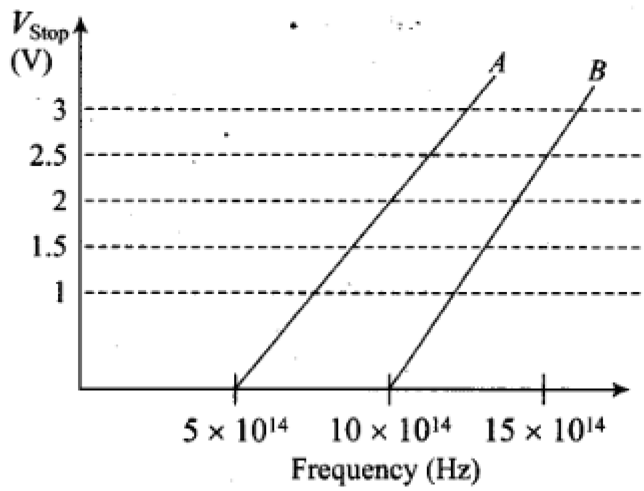
$$W = \int_d^\infty F_x dx = \int_d^\infty \frac{q^2 dx}{4 \times 4\pi\varepsilon_0} \frac{1}{x^2}$$

$$= \frac{q^2}{4 \times 4\pi\varepsilon_0}\left[\frac{1}{d}\right]$$

$$= \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4 \times 10^{-10}} \text{J}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (9 \times 10^9)}{(4 \times 10^{-10}) \times (1.6 \times 10^{-19})} \text{eV} = 3.6\ \text{eV}$$

**Question 27. A student performs an experiment on photoelectric effect, using two materials A and B. A plot of $V_{stop}$ versus v is given in figure.**




(i) Which material $A$ or $B$ has a higher work function?

(ii) Given the electric charge of an electron = $16 \times 10^{-19}$ C, find the value of $h$ obtained from the experiment for both $A$ and $B$.
Comment on whether it is consistent with Einstein's theory.

**Solution:**

**Key concept:** *Threshold frequency* ($n_0$) : The minimum frequency of incident radiations, required to eject the electron from metal surface is defined as threshold frequency.

If incident frequency $n < n_0 \Rightarrow$ No photoelectron emission

For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths between 200 and 300 nm), but for potassium and cesium oxides it is in the visible spectrum ($\lambda$ between 400 and 700 nm).

Here we are given threshold frequency of $A$

$$\nu_{OA} = 5 \times 10^{14} \text{ Hz and}$$

For $B$, $\nu_{OB} = 10 \times 10^{14}$ Hz

We know that

Work function, $\phi = h\nu_0$ or $\phi_0 \propto \nu_0$

$\Rightarrow$ $\phi_0 \propto \nu_0$

So,
$$\frac{\phi_{OA}}{\phi_{OB}} = \frac{5 \times 10^{14}}{10 \times 10^{14}} < 1$$

$\Rightarrow$ $\phi_{OA} < \phi_{OB}$

Thus, work function of $B$ is higher than $A$.

(ii) For metal $A$, slope $= \dfrac{h}{e} = \dfrac{2}{(10-15)10^{14}}$

or
$$h = \frac{2e}{5 \times 10^{14}} = \frac{2 \times 1.6 \times 10^{-19}}{5 \times 10^{14}}$$
$$= 6.4 \times 10^{-34} \text{ Js}$$

For metal $B$, slope $= \dfrac{h}{e} = \dfrac{2.5}{(15-10)10^{14}}$

or
$$h = \frac{2.5 \times e}{5 \times 10^{14}} = \frac{2.5 \times 1.6 \times 10^{-19}}{5 \times 10^{14}}$$
$$= 8 \times 10^{-34} \text{ Js}$$

Since, the value of $h$ from experiment for metals $A$ and $B$ is different. Hence, experiment is not consistent with theory.

**Question 28. A particle A with a mass $m_A$ is moving with a velocity v and hits a particle B (mass $m_B$) at rest (one dimensional motion). Find the change in the de-Broglie wavelength of the particle A. Treat the collision as elastic.**

**Solution:**

The laws of conservation of momentum and kinetic energy are obeyed because the collision is elastic.

By the law of conservation of momentum,

$$m_A v + m_B 0 = m_A v_1 + m_B v_2$$

$$\Rightarrow \quad m_A(v - v_1) = m_B v_2$$

And the law of consetvation of energy,

$$\frac{1}{2} m_A v^2 = \frac{1}{2} m_A v_1^2 + \frac{1}{2} m_B v_2^2 \quad \ldots(i)$$

$$\Rightarrow \quad m_A(v^2 - v_1^2) = m_B v_2^2$$

$$\Rightarrow \quad m_A(v - v_1)(v + v_1) = m_B v_2^2 \quad \ldots(ii)$$

Dividing Eq. (ii) by Eq. (i),

we get $\quad v + v_1 = v_2$ or $v = v_2 - v_1$ ...(iii)

Solving Eqs. (i) and (iii), we get

$$v_1 = \left(\frac{m_A - m_B}{m_A + m_B}\right) v \text{ and } v_2 = \left(\frac{2m_A}{m_A + m_B}\right) v$$

$$\lambda_{\text{initial}} = \frac{h}{m_A v}$$

$$\lambda_{\text{final}} = \frac{h}{m_A v_1} = \frac{h(m_A + m_B)}{m_A(m_A - m_B)v}$$

$$\Delta\lambda = \lambda_{\text{final}} - \lambda_{\text{initial}} = \frac{h}{m_A v}\left[\frac{m_A + m_B}{m_A - m_B} - 1\right]$$

**Question 29. Consider a 20 W bulb emitting light of wavelength 5000 Å and shining on a metal surface kept at a distance 2 m. Assume that the metal surface has work function of 2 eV and that each atom on the metal surface can be treated as a circular disk of radius 1.5 Å.**
**(i) Estimate number of photons emitted by the bulb per second. [Assume no other losses]**
**(ii) Will there be photoelectric emission?**
**(iii) How much time would be required by the atomic disk to receive energy equal to work function (2 eV)?**
**(iv) How many photons would atomic disk receive within time duration calculated in (iii) above?**
**(v) Can you explain how photoelectric effect was observed instantaneously?**
**Solution:**

According to the problem, $P = 20$ W, $\lambda = 5000$ Å $= 5000 \times 10^{-10}$ m, distance ($d$) = 2 m, work function $\phi_0 = 2$ eV, radius $r = 1.5$ A $= 1.5 \times 10^{-10}$ m

Now, Number of photon emitted by bulb per second, $n' = \dfrac{dN}{dt}$

(i) Number of photon emitted by bulb per second is $n' = \dfrac{p}{hc/\lambda} = \dfrac{p\lambda}{hc}$

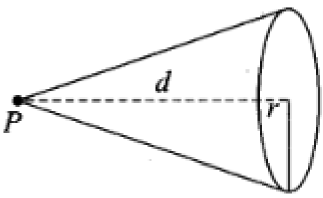
$$= \frac{20 \times (5000 \times 10^{-10})}{(6.62 \times 10^{-34}) \times (3 \times 10^8)}$$

$$\Rightarrow \qquad n' = 5 \times 10^{19}/\text{sec}$$

(ii) Energy of the incident photon

$$= \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{5000 \times 10^{-10} \times 1.6 \times 10^{-19}} = 2.48 \text{ eV}$$

As this energy is greater than 2 eV (i.e., work function of metal surface), hence photoelectric emission takes place.




(iii) Let $\Delta t$ be the time spent in getting the energy $\phi$ = (work function of metal).
Consider the figure, if $P$ is the power of source then energy recieved by atomic disc

$$\frac{P}{4\pi d^2} \times \pi r^2 \Delta t = \phi_0$$

$$\Rightarrow \qquad \Delta t = \frac{4\phi_0 d^2}{Pr^2}$$

$$= \frac{4 \times (2 \times 1.6 \times 10^{-19}) \times 2^2}{20 \times (1.5 \times 10^{-10})^2} \approx 28.4\text{s}$$

(iv) Number of photons received by atomic disc in time $\Delta t$ is

$$N = \frac{n' \times \pi r^2}{4\pi d^2} \times \Delta t$$

$$= \frac{n' r^2 \Delta t}{4d^2}$$

$$= \frac{(5 \times 10^{19}) \times (1.5 \times 10^{-10})^2 \times 28.4}{4 \times (2)^2} \approx 2$$

Now let us discuss the last part in detail. As time of emission of electrons is 11.04 s.

Hence, in this-problem, the photoelectric emission is not instantaneous.

(v) In photoelectric emission, there is an collision between incident photon and free electron of the metal surface, which lasts for very very short interval of time ($\approx 10^{-9}$ s), hence we say photoelectric emission is instantaneous.